PRIORITIES *in* PRACTICE

The Essentials of Mathematics, Grades 7–12

Effective Curriculum, Instruction, and Assessment

Kathy Checkley

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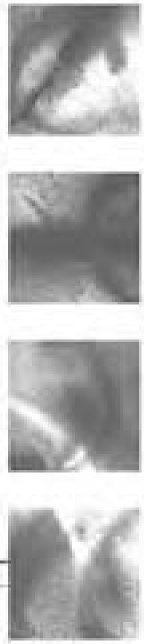

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Printed in the United States of America.

ISBN-13: 978-1-4166-0413-6 ISBN-10: 1-4166-0413-8
ASCD product no.: 106129 s9/06

Also available as an e-book through ebrary, netLibrary, and many online booksellers (see Books in Print for the ISBNs).

Quantity discounts for the paperback edition only: 10–49 copies, 10%; 50+ copies, 15%; for 1,000 or more copies, call 800-933-2723, ext. 5634, or 703-575-5634. For desk copies: member@ascd.org.

Library of Congress Cataloging-in-Publication Data
Checkley, Kathy.
The essentials of mathematics, grades 7–12 : effective curriculum, instruction, and assessment / Kathy Checkley.
p. cm. -- (Priorities in practice)
Includes bibliographical references and index.
ISBN-13: 978-1-4166-0413-6 (pbk. : alk. paper)
ISBN-10: 1-4166-0413-8 (pbk. : alk. paper) 1. Mathematics--Study and teaching (Middle school) 2. Mathematics--Study and teaching (Secondary) 3. Teacher participation in curriculum planning. 4. Educational leadership. I. Title. II. Series.

QA135.6.C524 2006
510.71'2--dc22

2006016138

13 12 11 10 09 08 07 06 1 2 3 4 5 6 7 8 9 10 11 12 13

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Acknowledgments

Many thanks! This book could not have been written without the kindness, patience, and generosity of all the teachers and educators interviewed.

Special thanks go to

• Jason Cushner, for reading the initial draft of this book and pointing out any errors or misconceptions.
• Cathy Seeley, who was always willing to take a few minutes to help clarify thinking.
• John Franklin, for his professionalism and constructive advice throughout this project.
• My John and Laura—with your support, all is possible.

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*These teachers received a 2003 Presidential Award for Excellence in Mathematics and Science Teaching.

Introduction

I advise my students to listen carefully the moment they decide to take no more mathematics courses. They might be able to hear the sound of closing doors.

—*James Caballero, "Everybody a Mathematician?"*
CAIP Quarterly

A new graduation requirement in the Prince George's County (Maryland) School District is keeping Wesley Brown busy.

The mandate stipulates that public school students who entered 9th grade in the fall of 2003 must take and pass the Maryland High School Assessments—which include the Algebra/Data Analysis Assessment. So Brown, a Region 1 mathematics resource teacher, spends his days working with teachers at four different high schools, helping them hone their instructional skills. "We have core learning goals, things our students must know to be successful," says Brown. "We want to help teachers be innovative" in helping students attain those learning objectives, he says.

Maryland is one of twenty states with mandatory exit exams—and five more states plan to phase in such tests by 2009, according to a Center on Education Policy report (Gayler, Chudowsky, Hamilton, Kober, & Yeager, 2004).

Brown thinks Maryland's Algebra/Data Analysis Assessment reflects a concern held by many educators and business leaders "that students don't graduate with the mathematics knowledge they need to be successful in continued schooling or their careers." Indeed, according to a report released by Public Agenda, many business organizations, including the

Unites States Chamber of Commerce, warn that American high school students "are not sufficiently skilled and knowledgeable about science and math in general" (Johnson, 2006, p. 1). When students lack these skills, notes the report, they put their future careers in jeopardy, as well as the ability for the United States to remain economically competitive with other nations.

Brown, therefore, is helping teachers help students learn their basic skills. "And algebra is the new basic," he says.

"Students are starting to realize that if they want better jobs and better opportunities, they need higher-level math," says John Bakelaar, assistant principal at Whitten Middle School in Jackson, Mississippi. And educators are starting to realize, he says, "that we need to make higher-level courses—Statistics, Calculus, and Trigonometry—available to all students."

Still, although most educators embrace more challenging math courses for all students and a majority of parents do believe that today's students should learn advanced algebra and calculus, most parents and students are generally happy with the status quo, according the Public Agenda's report. In fact, the number of parents who worry whether schools are teaching enough math and science has decreased since the mid-1990s.

Education, business, and government leaders, therefore, must now help parents and other stakeholders understand this paradox, says Public Agenda. These leaders must point out that failing to improve mathematics instruction for all threatens too many students' futures (*see p. ix*).

This push for public awareness comes none too soon for Nancy Foote, a mathematics teacher on assignment in Higley, Arizona. Indeed, when Foote attended a symposium held at film director George Lucas's Skywalker Ranch, she asked a congressman why mathematical illiteracy is okay. "I think there is a lot of lip service being paid to the fact that we, as a country, are falling behind in math," but not enough action is being taken, she says. For her part, Foote never misses an opportunity to tell her students that "it's not okay to not know math."

Harvey F. Silver and Richard W. Strong agree. "Math opens up career paths, empowers consumers, makes meaningful all kinds of data, from

Math Education: A National Imperative?

According to many educators, business leaders, and government policymakers, a sound mathematics education is key to a student's career success and to U.S. competitiveness. Even so, a prevailing ambivalence about whether all students need to become skilled in the subject may suggest otherwise.

Ask parents if they want their children to learn math, and they'll always answer yes, says Barbara J. Reys, distinguished professor of mathematics education at the University of Missouri. She adds that parents, however, will also say, "I wasn't good at math." Some parents also believe that not all students can learn mathematics and that "if they don't, it's okay," says Reys.

This vacillation may stem from parents' own unfavorable experiences with math in school, states Reys. It can also result from another kind of angst: Unlike with reading, "it's not too long before kids are studying math that's beyond what their parents studied. Parents are [then] out of the picture," she observes. Educators, too, have communicated mixed signals about how necessary it is for all students to learn mathematics, especially upper-level subjects like algebra and calculus. "Until recently, many people thought about mathematics as a discipline that is comprehensible to only a select, talented few," write Lynn T. Goldsmith and Ilene Kantrov in *Guiding Curriculum Decisions for Middle-Grades Mathematics*. "Instructional traditions paid little attention to helping students make sense of the mathematical ideas they encountered," the authors state (2001, p. 37).

Fortunately, the societal acceptance of poor mathematics achievement is waning. Dissatisfaction with poor math performance has "become intense and it is growing," write the authors of *Mathematical Proficiency for All Students*. "Every student now needs competency in mathematics," the RAND Mathematics Study Panel asserts (RAND Mathematics Study Panel, 2003, p. 2).

This goal is vital, because educational and career opportunities, as well monetary success, are directly linked to mathematics achievement, say researchers. Studies find, in fact, that

• Students who completed higher-level mathematics courses in high school were more likely to earn a bachelor's degree. A longitudinal study conducted by Clifford Adelman, a senior research analyst for the U.S. Department of Education, found that 8 percent of high school graduates with Algebra 1 under their belts earned a bachelor's degree by age 30. In contrast, 80 percent of those who completed Calculus in high school earned a bachelor's degree by age 30 (Adelman, 1999).

• More than half of workers earning more than $40,000 a year had completed two or more credits at the Algebra 2 level or higher, according to Anthony P. Carnevale and Donna M. Desrochers who analyzed data from the National Educational Longitudinal Survey (Carnevale & Desrochers, 2002).

• Taking higher-level math courses can boost a young person's earnings potential after high school, Heather Rose and Julian R. Betts report in *Math Matters: The Links Between High School Curriculum, College Graduation, and Earnings*. Rose and Betts found that after controlling for students' demographic, family, and high school characteristics, one extra course in algebra or geometry is associated with 6.3 percent high earnings (Rose & Betts, 2001).

Source: Adapted with permission from *Priorities in Practice: The Essentials of Mathematics, K–6*, (pp. 5, 10, 11), by K. Checkley, 2006, Alexandria, VA: ASCD.

basketball statistics to political polls to the latest trends in the stock market," they write in the foreword to *Styles and Strategies for Teaching Middle School Mathematics* (2003, p. 5).

For all these positive outcomes, however, Silver and Strong note a troubling reality: the longer a majority of students are in school, the less they trust in their ability to do math. In fact, the authors point out, more than three quarters of all students who graduate from high school don't believe that they are among the "special realm" of people who can be successful in a field that requires in-depth mathematics knowledge.

And that's a serious problem. "If we send an army of math-haters out into today's competitive global culture, we are short-changing millions of students by severely limiting their chances of future success," Silver and Strong warn.

One response would be to create an army of math-lovers—among students, teachers, administrators, and parents. The question is, How?

Luckily, we have some answers in this book.

• Teachers show how they are striving for equity. The stakes are high and every student needs to attain a higher level of mathematics understanding. The exemplary educators featured in this book share some proven strategies for helping them do so.

• Teachers share innovative lessons that address standards and help students see how math can be used in the world.

• Education experts discuss the research that influences how curriculum is developed and how instructional choices are made.

• Teachers and educators explore ways to vary instruction to meet their students' unique learning needs.

• Professional development experts, including teachers, discuss the kinds of learning experiences that teachers want and need.

We hope this book will help educators address the challenge of providing a sound mathematics education for all students.

Trends

I'm not out to convince anyone that calculus, or even algebra and geometry, are necessities in the hotel business. But I will argue long and loud that they are not useless ornaments pinned onto an average man's education. . . . To resolve any problem into its simplest, clearest form, has been exceedingly useful. It is true that you do not use algebraic formulae but . . . I found higher mathematics the best possible exercise for developing the mental muscles necessary to this process.

—*Conrad Hilton*, Be My Guest

Emphasizing Equity: Why *All* Students Must Learn Higher-Level Mathematics

Meet Gabrielle. She's a white, middle-class teen who attends a suburban high school with a primarily white, middle-class student body.

Meet Tamika. She's an African American, middle-class teen who attends an urban high school with an ethnically and socioeconomically diverse student body.

Both girls want to go to college. To qualify, both must perform well in college preparatory math classes. When both girls earn a *B* in their first such class, they are optimistic about their continued achievement.

Only one girl would ultimately remain in a college preparatory program, however. The story of what went wrong—and what went right—exemplifies the challenge of providing a high-quality mathematics curricula and effective instruction to all students.

Gabrielle and Tamika had similar aptitudes and attitudes toward math, but both needed to be supported in ways that extended beyond instruction. The girls' academic achievement was influenced by how each school's math department was organized, notes Ilana Horn, assistant professor of mathematics education at the University of Washington in Seattle. Horn contrasted the two girls' experiences in "Why Do Students Drop Advanced Mathematics?" (2004).

"Gabrielle found herself uncomfortable being a junior in a class filled with freshmen and sophomores." She was reluctant to ask questions in class, not wanting to look "dumb." When Gabrielle could only offer a partial solution to a problem, the teacher failed to guide her toward the answer and, instead, moved the lesson forward when another student offered the correct answer. "Subsequently, Gabrielle's participation and forthrightness with questions diminished," Horn writes. "Gabrielle eventually stopped doing her homework and ultimately earned a *D* in the course" (p. 63).

Tamika also found the advanced math curriculum to be far more challenging than her earlier algebra class. Her geometry class moved at a faster pace, and students were expected to be more responsible for their learning. "In the algebra classes, teachers kept students inside during lunch or after school to complete any unfinished homework," writes Horn (p. 63). In the geometry class, if students needed help, they needed to seek it out. Tamika did not seek help. Although she received a *B-* in the first half of the class, she ended the course with a *D*, earning credit but not a promotion to the next math class. She retook the second half of the class, but despite a more concerted effort on her part, Tamika did poorly on a final major test and, again, earned a *D*.

The two stories end in decidedly different ways. When Gabrielle performed poorly, "she retreated to the remedial track for her senior year," writes Horn (p. 63). Tamika, on the other hand, was in a school that had no remedial track. "Her geometry teachers evaluated her work and decided to allow her to attempt advanced algebra, the next class in the curriculum. She earned a *B* in the first semester and a *C* in the second semester of advanced algebra. In her senior year, Tamika earned a *B*+ in precalculus, a course that exceeds college entry requirements," Horn writes (p. 63).

Among the factors that led to Tamika's success was the school's philosophy of building high expectations into the school's mathematics curriculum. Without a remedial track option, teachers were compelled to ensure Tamika would succeed, Horn explains.

Success stories like Tamika's may lead one to conclude that tracked high schools should be a thing of the past. But, Horn states, although "morally we should do it," many teachers lack the resources to do it right. "We have to address the structures of school," she says. School leaders need to overhaul schedules that give students more opportunities to retake courses they find difficult, for example, and free up more time for teacher collaboration and professional development. Horn adds that school leaders and teachers also have to be aware of—and address—social status issues.

"The tracks become social spaces," says Horn. "Kids with certain identities" prefer to remain with their peers, and they may "affiliate academic success with being 'white' or 'nerdy.'" Moving to a college preparatory track can be socially risky for these students, she explains. Horn recalled an African American student from a diverse urban—and tracked—school where she once taught. He was "super bright," but hadn't enrolled in an algebra class. "I remember saying during a parent-teacher conference that he needed to be in an algebra class. But he would not do it," says Horn. His mother eventually enrolled him in a private school where the prevailing culture didn't thwart his potential. His mother, Horn says, "didn't want him to be in a school where he couldn't achieve."

And promoting achievement in more rigorous math courses has become a central focus of educators interested in reforming K–12 mathematics. Indeed, equity is one of six core principles included in the *Principles and Standards for School Mathematics* (*PSSM*), published by the National Council of Teachers of Mathematics (NCTM, 2000).

"Excellence in mathematics education requires equity—high expectations and strong support for all students," the *PSSM* authors write.

"All students, regardless of their personal characteristics, backgrounds, or physical challenges, can learn mathematics when they have access to high-quality mathematics instruction" (NCTM, 2000, p. 2).

A Laudable Goal but Hard to Achieve

Still, as Horn illustrates in her examples, and as other researchers find, providing students with access to high-quality mathematics is much easier said than done.

Time and quality factors, for example, play a big role in whether students of color or students in high-poverty schools receive the kind of instruction supported by NCTM, report Celia K. Rousseau and Angiline Powell in a study published in the *High School Journal* (2005). Rousseau and Powell, both education professors at the University of Memphis, compared the instructional approaches of two Algebra 1 teachers.

In one classroom, students work in groups to explore linear functions through a hands-on experiment, while the teacher walks around the room, asking questions to guide students' thinking such as, What's unusual here? What do you notice?

In another classroom, the teacher distributes test preparation booklets and defines, with limited student input, such terms as "opposite," "additive inverse property," and "reciprocal." The students then spend the rest of the class answering the multiple choice and fill-in-the-blank questions in the test preparation book.

In later observations of both teachers—one, a suburban 8th grade algebra teacher and the other, an urban high school teacher—Rousseau and Powell found that each used a mix of such traditional and reform-guided instructional approaches. Still, the authors note in "Understanding the Significance of Context: A Framework to Examine Equity and Reform in Secondary Mathematics": One teacher does appear to be closer to the reform end of the continuum than the other. Why?

Perhaps, not surprisingly, planning time was seen as a key factor in influencing teachers' decisions about how to teach, write Rousseau and Powell. "There is no time in the day to search, plan, and write tasks like these each school day," noted one teacher (p. 26). Large class size was another factor: although the suburban teacher had smaller classes (averaging 15 students), the urban teacher had 35 students in one of her classes and was, therefore, reluctant to use cooperative learning and other discovery-based activities.

Perhaps a much larger issue, however, was the influence of the state-mandated test. Students in both schools took the same test. Yet, the suburban teacher "appeared to feel little pressure to focus on test preparation," confident that his students would pass the test "even without explicit efforts on his part to prepare them," write Rousseau and Powell. Conversely, the urban teacher "felt substantial pressure to prepare her students." Indeed, the teacher would stop instruction on new content "two full months before the state exam" to begin test-prep work, the authors found (p. 27).

Why such a difference in attitudes toward the test? The answer lies "within the broader context," Rousseau and Powell state (p. 28). In the urban school, more than half the students tested failed to meet the level of proficiency for algebra. The school was also in danger of failing to meet adequate yearly progress. In the suburban school, on the other hand, more than half of the students tested reached the advanced level for algebra.

Interestingly, both teachers had participated in a professional development project that focused on teaching algebra for understanding. It would be easy to conclude, therefore, that the professional development effort had been successful with one teacher but not the other, but Rousseau and Powell point out that this explanation is too simplistic. What's more likely, they say, is that a combination of factors, including larger class size and pressure for students to perform well on standardized tests, served to limit one teacher's ability to change. More professional development probably wouldn't help the urban teacher much, the authors note. What that teacher and her students need is a system "that can better support all teachers as they seek to change" (p. 30).

Hard to Achieve, but a Necessity

And change they must, because "most kids are underserved by the way we teach math now," says Horn. She adds that, as an advocate for teachers, she really can't blame them much: "Most of our high school teachers see 150 to 180 students a day." So, she calculates, if a teacher takes one minute a day to review each student's work, he'd spend almost three hours on the task. Yet most schools, she points out, allot teachers a single, 55-minute planning period during the school day. "We give teachers too many students and not enough time," Horn states. "Teachers aren't being given the kinds of things they need to reach kids."

Innovative scheduling can help. In Tamika's de-tracked school, for example, teachers devised a 4 x 4 block schedule—"kids had four 90-minute classes per day." That schedule allowed teachers a 90-minute planning period, says Horn. Another school used state dollars to achieve a 20:1 student-teacher ratio for its first-year algebra class. The result was that teachers could have one-on-one conferences with students and follow-up meetings with parents, she explains.

Still, it's not enough to just expand planning and class time, says Horn. Block scheduling competes with a widely-held notion that mathematics consists of mastering procedures, she states. "Teachers have to learn how to teach for depth, not for coverage; they have to see mathematics more broadly—as justifying things, making representations, showing understanding and concepts in different ways," says Horn. And that's a "real challenge" for many teachers, she notes.

NCTM's president, Cathy Seeley, agrees. Teaching for understanding "calls for a strong knowledge of math and a lot of support for teachers during implementation," she says. "It takes some time before teachers become as adept as you might like."

Time, however, may not be on our side, say educators who believe a fast-growing global economy dictates a more aggressive reform timetable.

The proof is in the news headlines, suggests William Schmidt, a distinguished professor at Michigan State University. Consider this short sampling:

• “Competing with India and China”—*U.S. News & World Report*, October 10, 2005

• “Michigan’s Job No. 1: Recovery; Looking to Toyota for a Helping Hand”— *The New York Times*, December 27, 2005

• “Filling the Engineering Gap”—*Business Week*, January 10, 2006

• “Math Will Rock Your World: The New Era of Numbers”—*Business Week*, January 23, 2006

Schmidt admits to a sense of urgency when he reads such stories—or attends meetings with business leaders who don’t mince words. “I was at a meeting with Intel and one of the executives said the nation is at a tipping point,” he says. If the citizenry is not educated well enough in math and science, “if we don’t provide our workforce with that background, it will affect our country’s future and the kinds of jobs students can get.” Especially now, says Schmidt, when technology has enabled “anybody in the world” to do the kind of knowledge-based work that predominates in the global economy.

Adding to Schmidt’s distress is his contention that huge inequities in access to effective math and science instruction are creating “an undereducated underclass.” Improving the knowledge base of more young people is the only way to reduce social problems, such as crime and welfare dependency, Schmidt asserts. “Unless we address this problem and quit failing our kids in mathematics and science, major corporations will move jobs elsewhere and our students will be left with service jobs,” he warns.

“People are getting scared,” agrees Nancy Foote, a secondary math teacher in Higley, Arizona. “They see jobs being outsourced and technology advancing.” People are worried that today’s students “aren’t equipped to handle” the competition, she says.

Toward Equity Through Early Exposure

To better equip students, make sure they attain a reasonable knowledge of algebra and geometry and even some calculus, Schmidt advises. The tools students need today are different from those needed yesterday, he notes. "What is fundamental now is not what it was years ago—the level of required knowledge has ratcheted up."

Teachers, therefore, should emphasize problem solving and the other NCTM process standards to help give students the confidence they need to take on those advanced courses, adds Foote. "We can't just give them information—there's too much of it," she states. "We have to give them the tools [such as the ability to problem solve and use critical thinking skills] to process and apply that information."

Don't wait until high school to do so, however, says Don Karlgaard, a 9–12 math teacher at Brainerd High School in Minnesota. "We have a national movement to educate all kids well," he says. But, in order for all students to understand a subject like geometry, for example, "they need to start learning it in kindergarten."

Kristen Scanlon, director of federal programs for the Fayetteville (Arkansas) Schools, agrees. "The earlier they start, the better," she states. "It's about exposure." In Arkansas, where students must take four years of mathematics to graduate, there is an emphasis on introducing formal algebra in middle school, Scanlon says students take Algebra 1 in 8th grade, Geometry in 9th, Algebra 2 in 10th grade, and so on.

Indeed, educators in many school districts across the United States are grappling with how to best align K–12 mathematics programs and expose more students to more rigorous mathematics. "There is a debate about how to fit algebra into the overall middle-grades curriculum," write Lynn T. Goldsmith and Ilene Kantrov in *Guiding Curriculum Decisions for Middle-Grades Mathematics*. "It is important to be thoughtful about what we mean by 'algebra' and what algebra instruction should look like in the middle grades," (2001, p. 44).

According to Goldsmith and Kantrov, formal algebra courses, such as those in high school, typically "present algebra in abstract terms and focus on rules for manipulating symbols and solving equations" (p. 44). Is this the best approach to take with middle-grade students? the authors ask. Some students may have a "facility with abstract reasoning," Goldsmith and Kantrov note, but others may not (p. 45).

Differentiate between "algebraic thinking" and "symbol manipulation"—and then look to the NCTM *Standards* for guidance, Goldsmith and Kantrov suggest. “The *Standards* recommend that algebraic thinking be an integral part of the curriculum from elementary years onward.” In the beginning, students can explore meaningful patterns and relationships. Then, in middle and high school, students can move toward “increasingly formal representations and more complex manipulations and transformations,” the authors write (p. 45).

Still, if middle school students take courses that promote algebraic thinking, they may not be “as facile with manipulating equations as those whose algebra classes focus heavily on [such] skills,” Goldsmith and Kantrov caution (p. 45). Therefore, communication between middle and high school teachers about what students can do well and what they still need practice with becomes crucial.

A more sophisticated use of data is also helpful in determining students’ readiness for advanced mathematics, says Jeffrey Chaffee, the math curriculum enhancement teacher for Remington Middle School in Franklin, Massachusetts. In his district, data collected from pretests—administered before each unit beginning in 6th grade—allows teachers to differentiate instruction. Some children already know the math that will be covered in that unit, “even before they’ve been taught the material,” says Chaffee. These students are then placed in an enrichment group. “They still cover material, but we can do harder problems with them and have deeper discussions,” he says.

Such pretests and placements continue throughout 6th grade. And Chaffee has found that the process reveals each student’s unique mathematical strengths. After students take another unit pretest, “you may have an entirely different group of kids in the enrichment class,” he says. “Sometimes the groups will be as big as twenty students, sometimes as small as six or seven—it really varies by topic.”

At the end of the year, Chaffee and the two other math specialists in the district analyze students’ pretest and state test scores to determine which students qualify to skip 7th grade math and go straight to algebra. Somewhere between 7 and 15 kids can pick up algebra quickly enough that they don’t need the pre-algebra covered extensively in 7th grade, Chaffee explains. Then, “when those kids become 8th graders, they’re ready for geometry,” which Chaffee teaches.

The opportunity for such acceleration doesn't stop with middle school. With algebra and geometry under their belts, these students can take Algebra 2 as freshmen, Calculus as juniors, and Advanced Placement Statistics as seniors. "We're also trying to get our high school to offer a 2nd year of calculus," Chaffee states.

Educators in the Franklin, Massachusetts, district have found an innovative way to use test data to meet the individual needs of their students. That shows, says Seeley, that "if we do this well, we can go further than we used to go. We can ask more sophisticated kinds of questions and allow kids to think more deeply."

Acceleration for All?

The students in Chaffee's district showed an aptitude for advanced mathematics, and their needs were met accordingly and appropriately. One of the more challenging—and contentious—questions facing reformers today, however, is how to best deliver a rigorous curriculum to *all* students, including those who struggle with math.

Seeley is often asked that question. Some teachers, for example, believe that it just isn't fair to require low-achieving students to take higher-level math courses. These students simply won't be successful, the teachers say, and requiring them to take higher-level courses deprives them of the practical mathematics they will eventually need. Seeley offers a three-fold response to such concerns.

First, she says, although higher-level math is seen as abstract, the math supported by NCTM's *Standard*s includes real-world applications. Seeley adds that the way a knowledgeable teacher teaches math "makes all the difference in the world in helping students with different levels of experience and expertise deal with even the most challenging problems."

Second, it's difficult to know which students might want to attend college long after they leave the high school classroom. After reflecting on the tracking practices that were in place when she started teaching in the 1970s, Seeley concluded that "the notion that some students were college-bound and some were not is outmoded." Some students may decide to return to school after working a few years, she says. If these students have studied only remedial or low-level math, "they will have to spend a year or more catching up."

Third, most experts agree that the majority of students in low-level mathematics courses are from a different demographic than most students in advanced classes. "Do our advanced calculus classes reflect approximately the same racial and socioeconomic balance as our lower-level classes?" Seeley asks, adding that students who come from diverse backgrounds and "who bring different levels of success in previous mathematics classes" represent "untapped potential" and might not necessarily have less ability to succeed. Therefore, providing *all* students with more high-quality (and high-level) mathematics, gives all students more options for their futures, she notes.

Research appears to support Seeley's contention. Carol Corbett Burris, principal at South Side High School in Roxville Center, New York, wrote her dissertation on what happened when the middle school in her district adopted universal acceleration. The former Spanish teacher, who had seen positive results when her school blended its two 8th grade Spanish tracks, says she "had no idea" what she would find when she embarked on her study.

What she found were myriad studies documenting the damaging affects of the traditional remedial curriculum on math performance and an equal number of studies that strongly supported alternatives, including acceleration. Burris's evaluation of what occurred at South Side Middle School provided further evidence of the need to offer a rigorous mathematics program to all students.

In an *Educational Leadership* article she coauthored with Jay P. Heubert and Henry M. Levin, Burris describes the multiyear program to eliminate tracking at South Side Middle School. "Starting in 1995, all students entering 6th grade . . . took accelerated math in heterogeneously grouped classes and prepared for taking the Regents exam when they reached 8th grade," write Burris, Heubert, and Levin (2004, p. 69).

To prepare students for the accelerated algebra course, usually reserved for 8th grade high achievers, the middle school teachers worked together to revise and streamline the regular 6th and 7th grade math curriculum, eliminating redundancies. Then, "the school supported struggling learners in three ways," write the authors (p. 69):

- Special support classes met every other day in addition to the regular class meetings.
- Teachers, as part of their work contract, provided after-school help four days a week.
- The school supplied general resource support, such as providing supplementary materials.

It was a five-year reform effort, but by the year 2000, middle school math teachers said "they had no interest in returning to the previous tracked system," Burris and her colleagues write (p. 69).

In looking at the numbers, the teachers' enthusiasm for acceleration is well understood. "By every measure, students benefited from studying accelerated math in heterogeneously grouped classes," write the authors. "The research documented a statistically significant increase in the percentages of all students who took math courses beyond Algebra 2 in high school," state Burris, Heubert, and Levin (p. 70).

Other numbers of significance:

- The percentage of students from low socioeconomic backgrounds completing trigonometry before high school graduation increased from 32 to 67 percent.
- African American and Latino students completing trigonometry before graduation increased from 46 to 67 percent.
- Initial low achievers completing trigonometry before graduation increased from 38 to 53 percent, with average achievers in that category increasing from 81 to 91 percent.
- Even the percentage of initial high achievers completing trigonometry before graduation increased—from 89 to 99 percent.

The rates at which each group took Pre-Calculus and Advanced Placement Calculus also increased.

Perhaps the "most remarkable outcome" of implementing accelerated math at South Side Middle School was that, despite initial predictions, "we had no decrease in the performance of high-achieving students," says Burris. "If anything, we had higher achievement." Burris believes it was because the strategies used to reach lower-achieving students benefited higher-achieving students as well. That trend continues today. "We have teachers who love kids and want to be successful with all kids," she says. "In short, they've become better teachers and try harder."

Helping Students Who Struggle

Providing adequate support is a key reason accelerated math is successful at South Side Middle School. The schedule was modified so a full period is provided for any student who needs support. "Kids could go in and out as needed," Burris explains. "Some kids are there because they *have* to be there—others simply elect to be there."

Educators in the Glendale (Arizona) Union High School District have long understood the need to offer "some kind of intervention" to ensure students are prepared to handle a high school mathematics curriculum, says Warren Jacobson, associate superintendent of curriculum and instruction.

The district's nine high schools serve roughly 14,500 students in grades 9–12. Students transfer in from two elementary school districts, each with its own governing board and instructional philosophy. And, while students may have had different math experiences in their elementary schools, as soon as they enter Glendale Union, they all have a common challenge: they must pass Arizona's Instrument to Measure Standards (AIMS) test in the fourth quarter of their sophomore year. "Time is short," says Jacobsen. "We have less than four semesters to get them up to speed in algebra." In response, the summer pre-algebra project was instituted. The project has, of course, evolved over the past 20 years, Jacobsen notes, and it now sports a new name: Project SHARP—the Summer High School Algebra Readiness Program.

The three-week program addresses nine skill areas (*see p. 14*), students participate in two field trips to local businesses and museums, and those who successfully complete the program earn a one-quarter elective credit, which they can apply toward graduation. Cory Shinkle, the district's math coordinator, begins recruiting for Project SHARP in January and February, visiting each of the feeder schools to encourage students to participate and to hire teachers.

Marketing is critical, Jacobson observes. Although the program is free, it is voluntary, and the district doesn't always reach the targeted students—those who are weak in math. Shinkle, therefore, emphasizes the hands-on, interactive nature of the instruction—students will learn the math but have fun doing so.

"Many of the activities in Project SHARP are student focused," says Ken Cameron, the district director of research and evaluation. "It's not a traditional high school math class—the teachers even use games in their classrooms" (*see p. 15*). The students also go on field trips designed to show how mathematics is used in real-world settings. For example, one trip is to a pizza parlor where students make their own pizza. "It fits with the standards," says Shinkle. Students find the area of a pizza and determine what ingredients to order, along with the quantity.

Project SHARP Program Outcomes

Students will learn to

1. Apply fundamental arithmetic skills.
2. Demonstrate proficiency in using order of operations.
3. Demonstrate understanding of algebraic definitions.
4. Demonstrate applications and properties of integers.
5. Evaluate and simplify algebraic expressions.
6. Convert words to symbols.
7. Solve linear equations in one variable.
8. Demonstrate arithmetic of polynomials.
9. Apply letter writing and notebook organization.

Project Sharp has been successful in helping students make a smooth transition into high school algebra, says Jacobson. In 2004–05, for example, close to 100 percent of students who attended achieved each of the nine program outcomes; that group did reasonably well on the AIMS Math test (43 percent of students were successful); and Project SHARP participants scored higher on the Stanford 9 Math test (post-project participants, moving from grade 8 to grade 9, were in the 54 percentile rank, up from 43).

Teachers also fared well through their participation, Shinkle notes. "The teachers love teaching the program. It's a team-teaching environ-

The Pyramid Game

One of the games most popular with students in Project SHARP is the Pyramid Game, says Cory Shinkle, the district math coordinator for the Glendale Union High School District in Arizona. "It's just a blast," he observes.

Here's how it works:

• Students get into four teams and are given a worksheet of problems to solve.

• Students must complete a set of problems in a given amount of time.

• The teacher selects a group to start. If the group arrives at the correct solution, they get to pick a numbered box (see *The Pyramid Game, p. 16*.)

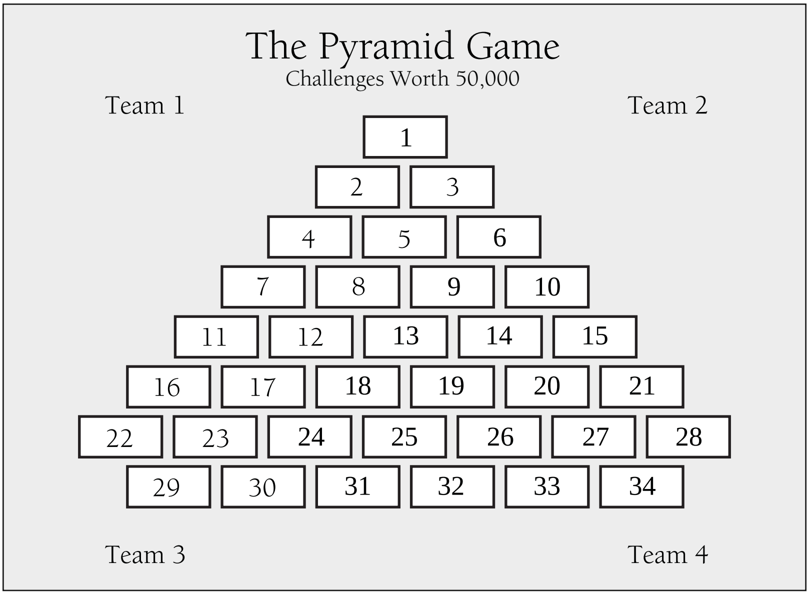
• At the end of the game, tally each team's points. Those with the highest point value at the end of class wins.

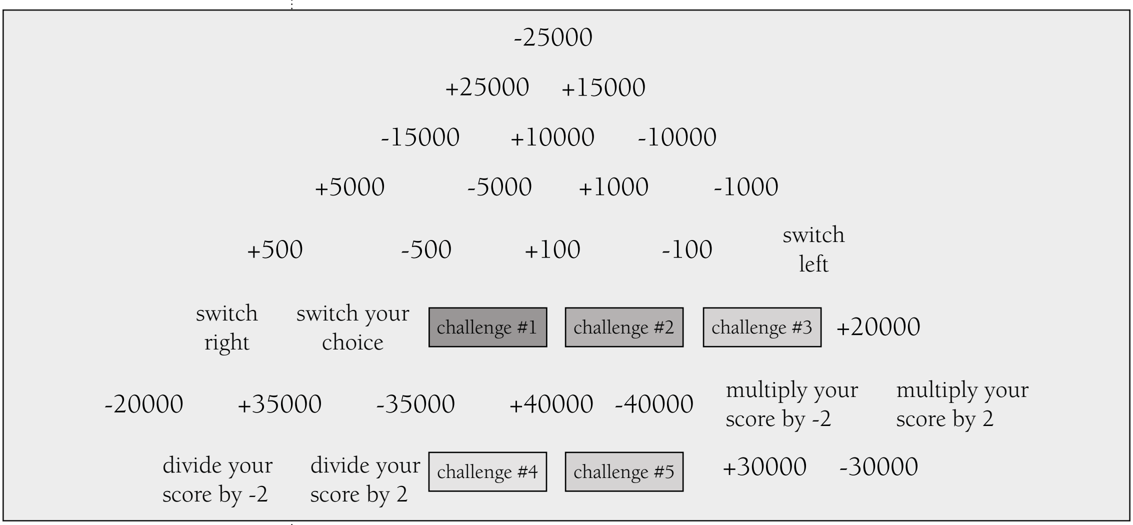
There are also five challenges on the board, so if students choose that numbered box, they must comply with the instructions to earn the points. Teachers create the challenges, which have included having students sing *Mary Had a Little Lamb*, completing a certain number of push-ups, and playing a word scramble: Given POLYNOMIAL, if you have so many words in 30 seconds, you get the points.

"Students get very competitive," says Shinkle. And, because it's a team competition, "stronger students will assist the weaker ones. It's one-on-one tutoring, which really helps each student understand the concept better."

ment. Then, after teaching from 8 a.m. to 12:30 p.m., the teachers get together and collaborate for 2 hours. Teachers have said that they've learned some of their best teaching strategies and techniques during those two hours, for which they get paid," he says.

"Teachers recognize Project SHARP as a valuable professional development opportunity," Cameron agrees. "Some teachers have stated that





they will take the strategies they learned in Project SHARP and employ them in their classrooms." The project also gives teachers from schools that feed into the district "an opportunity to see what's expected in math at the high school level," he says. Conversely, high school teachers "see what's occurring in the 7th and 8th grade math classes."

Still, one of the best outcomes, says Shinkle, is the self-confidence in students the project inspires. “These students learn they can enjoy math. For many, it’s the first time they’ve passed math in years. It’s student centered, so students are teaching each other, students are helping each other—and they’re doing better in math.”

Reflections ◆ ◆ ◆

• **Keep it relevant.** All students benefit when the curriculum helps them see how mathematics is relevant to their daily lives. If the activities are fun—and making pizza would qualify—it’s an added advantage. Some research has found that when students are amused, they let go of any stress that might have impaired their ability to participate and learn. Humor holds students’ attention, and students retain more of what they learn (Goodman, 2002).

• **There is a future in numbers.** As William Schmidt points out, the proof is in the headlines: knowing math is key to competitiveness in the global marketplace. Indeed, in the January 23, 2006, issue of *Business Week*, it’s reported that there has never been a better time to be a mathematician (Baker & Leak, 2006, p. 54). Industries look to mathematicians to help them sort through their “swelling oceans of data” (p. 56). From careers in consulting to advertising to police and intelligence work, people with an aptitude for math will be secure in tomorrow’s job market. This, of course, makes achieving equity an even more pressing concern.

• **Access = Equity.** As the educators in this chapter have pointed out, equity will not be achieved if all students are not given equal access to high-quality, rigorous mathematics instruction. There are several examples of programs that work in this chapter. For more examples of educators who are successfully closing gaps in achievement by addressing equity issues, take a look at the November 2004 issue of *Educational Leadership*, which focuses on that topic. You may also wish to do a search on the achievement gap in ASCD’s online store (www.shop.ascd.org). You will be able to browse the books, audiotapes, video programs, and other materials that ASCD offers to address this important issue.

2 Considering Curriculum

It isn't that they can't see the solution. It's that they can't see the problem.

—G. K. Chesterton (1874–1936)

Choosing curricular materials to support a mathematics program has never been easier—or harder: easier because there are so many options available today; harder for the same reason. It's a classic, glass half-full or glass half-empty scenario.

"When I taught 20 years ago, textbooks were a lot alike, but, now, there are a lot of different options," says Barbara J. Reys, distinguished professor of mathematics education at the University of Missouri. "If you want to keep a traditional approach, there are books for that. If you want to do something different, you may push to adopt those kinds of materials."

The key, of course, is that schools and districts must choose, and choose wisely. "There is no one-size-fits-all textbook or set of curriculum materials," observes Reys, adding that care must be taken when selecting materials because they have "a big impact on what kids get to study and how teachers teach."

According to Reys, textbooks can be "change agents." Therefore, it's particularly important that all stakeholders are in agreement about what direction their mathematics program takes. As she and coauthors Robert E. Reys and Oscar Chávez write in "Why Mathematics Textbooks Matter" (2004), it takes a strong team of people to select materials that will help a school or district remain true to its vision. This team includes the principal, teachers, and parents, write the authors. "Each has a special role in the process" (p. 66):

• **The principal** provides leadership and establishes clear and specific learning goals in mathematics. "He or she not only facilitates the selection of good textbook materials, but also ensures that teachers are kept informed about current mathematics curricular alternatives," write Reys, Reys, and Chávez (p. 66). The principal should also ensure teachers have time to meet and review textbooks, attend workshops on promising curricula, and test materials in their classrooms.

• **The teacher** supports all students in learning mathematics. Teachers must also be open to change "if current techniques and resources do not meet this challenge" (p. 66).

• **The parent** becomes knowledgeable about the mathematics topics that his children will be learning. Parents should attend school math nights and engage in "a specific mathematical activity drawn from the textbook," write Reys, Reys, and Chávez. This will help parents "better understand and appreciate the level of mathematical activity that their children will regularly encounter in the classroom" (p. 66).

With roles assigned and understood, the authors recommend that team members ask the following questions to effectively assess the materials:

• What key mathematical ideas in each content strand should each grade level address?

• How does the content of the textbook align with these key mathematical ideas?

• What types of activities does the textbook provide? Are students challenged to think and develop understanding, or are they simply shown how to work some exercises and then asked to practice procedures? Will these activities engage students in mathematical thinking and activity?

• Is there a focus on mathematical thinking and problem solving? Are students expected to explain "why"? Does the textbook encourage students to explore "what if" questions and to offer and test conjectures? (p. 65)

The strategy sounds simple enough—in theory. In reality, however, arriving at consensus is seldom a pain-free process. Part of the problem is that, despite having a set of national standards that guides the direction of a mathematics program, and despite the mounting evidence that our students' future job security depends on attaining a sound mathematical foundation, there is still disagreement about what math—and how much of it—students need to know.

Still a Vision Thing

"As a nation, we don't have a consensus of what should be taught," says Reys. What students learn varies dramatically from state to state and, as a result, she says, "teachers have an enormous set of materials that they need to choose from." In Reys' opinion, such variety leads to poor quality textbooks that fail to provide a "thorough treatment of concepts at particular grade levels"—unlike textbooks used in Japan and Singapore, whose students routinely perform well on international exams. This is one reason for Reys' belief that the United States needs a national curriculum. "But I don't think I'm in the majority," she observes.

William Schmidt is definitely in her camp, however. "I really think that the vast majority of the problems that we're confronted with are derived from the fact that we haven't addressed that issue," says Schmidt, a distinguished professor at Michigan State University who helped direct and coordinate U.S. participation in the Third International Mathematics and Science Study (TIMSS). "We have worried about curriculum alignment, and alignment of assessments, and teacher preparation, and we keep coming back to 'why aren't things better?' " he observes. These problems "don't exist" in places where "there is a clear articulation of things at a national level," Schmidt maintains.

And Schmidt isn't sure the NCTM *Standards* offer a "de facto" curriculum, as some educators claim. Certainly, there is agreement in the "broadest sense," he concedes, but data show that "as you go from district to district, there is still a fair amount of variation across states about what it is we expect kids to learn across grade levels."

Most states are "happy with the way things are," notes Reys. And maybe someday, she says, technology will allow the publishing industry to tailor textbooks that align with each state's curricula (it's too cost prohibitive today). Such advancement "would serve us well in terms of curriculum frameworks and alignment of state assessments." Until then, Reys wonders if curricular materials and textbooks should be more closely aligned to national assessments, such as the ACT Assessment and the Scholastic Assessment Test (SAT). These assessments aren't linked to any one state's standards, she points out, but many colleges use students' performance on these tests to help them determine admission to their programs.

Of course, Reys adds, if there is a debate over whether the United States should have a national curriculum, a second debate is over who will get to write it. One option, says Schmidt, is to convene a panel of experts for the discipline—mathematicians, educators, teachers, and people representing the business community—and work together to determine what knowledge and skills are required for success after high school. "I've been involved in that process in Achieve and it worked here in Michigan," Schmidt states. "Initially, people were quite disparate in their opinions, but agreements can come through collaboration."

Still, the road to a national curriculum can be a bumpy one, as educators "down under" have discovered.

About Achieve

Achieve, founded in 1996, was created by state governors and business leaders. The bipartisan, nonprofit organization helps states raise academic standards, improve assessments, and strengthen accountability.

Through the American Diploma Project, Achieve has developed benchmark standards that describe the specific math and English skills high school graduates must have if they are to succeed in postsecondary education and high-performance jobs.

More information is available online at www.achieve.org.

In Search of a National Curriculum

Creating a national curriculum has been a recurring initiative in Australia since the early 90s. But, "designing a national curriculum is far from easy," Kevin Donnelly writes in *On Line Opinion* (2003, para. 4). And, after initial efforts were unsuccessful, education ministers are moving a bit more cautiously this second time around, says Donnelly, executive director of the Melbourne-based consulting group Education Strategies.

Education leaders want to avoid adopting a curriculum that could be described as mediocre—as the first set of standards were characterized, writes Donnelly (2003, para. 6), who was commissioned to analyze Australia's primary mathematics, science, and English curricula. "Vital in achieving this is a willingness to look outside territorial boundaries and to identify the characteristics of better performing countries and to adopt best practice," Donnelly writes in the resulting report, *Benchmarking Australian Primary School Curriculum* (2005, p. 3).

Donnelly notes that Australian students haven't significantly improved their performance since the 1994–95 TIMSS and "are now being outperformed by countries that were once below us in ranking" (p. 3). He faults a curriculum that attempts to cover too much ground, adding that the "principal shortcoming" of Australia's curriculum documents is "their failure to specify clearly and in sufficient detail what is important to teach and what students are expected to understand, know, and be able to do," unlike the materials from other countries he analyzed (*see p. 23*).

To remedy the problem, Donnelly has recommended a "syllabus" approach to curriculum development: "Teachers are given, at the start of the year, a clear, succinct and easy-to-follow syllabus of what should be taught," he writes in *On Line Opinion*. Such an approach is similar to that taken in Japan and Singapore, notes Donnelly, who gives the curricula in those countries high ratings for being detailed, unambiguous, and measurable. Consider, for example, Donnelly's comments about Japan's mathematics learning program:

> Like Singapore, Japan has long been one of the high achievers in mathematics [on] the TIMSS and other tests. High expectations, government approved textbooks, [and] sustained 'lesson study' by all teachers at local school and district level work together to support success. The syllabus document is a model of concise and clear expectations about what children are expected to understand, to know and to be able to do. (p. 52)

Strong and Weak Descriptors		
AREA COVERED	**STRONG**	**WEAK**
Multiplication Year 3	**California: Mathematics Content Standards Grade 3** • Students calculate and solve problems involving addition, subtraction, multiplication, and division. • Memorize to automaticity the multiplication table for numbers between 1 and 10. • Solve simple problems involving multiplication of multidigit numbers by one-digit numbers (*3,671 x 3 = _*)	**Tasmania: Derwent Primary School Mathematics Program Year 3** • Addition, multiplication, subtraction, and division are interrelated and provide powerful ways for operating with numbers. • There are useful patterns in the multiplication and addition tables.
Division Year 4	**Japan: Mathematics Program Grade 4** • Children should deepen understanding of dividing integers, should be able to do these calculations reliably, and should extend the ability to make use of them appropriately. • To consider how to divide when the divisor is a one- or two-digit number and the dividend is a 2- or 3-digit number, and to understand that these calculations can be done on the same basis as more basic calculations. To understand how to do these calculations in column form. • To be able to divide reliably and to make use of division appropriately. • To investigate relations between dividend, divisor, quotient, and remainder, and to represent it in the following expression: Dividend = Divisor x Quotient + Remainder. • To investigate the properties of division, and to use the results to formulate multiplication tables, check the results, and so forth.	**New South Wales, Mathematics K–6, Syllabus Stage 2** • Uses mental and informal written strategies for multiplication and division. • Develops mental facility for number facts up to 10 x 10. Interprets remainders in division problems.

Source: From *Benchmarking Australian Primary School Curricula,* by K. Donnelly, 2005, October. © Commonwealth of Australia 2005. Retrieved January 23, 2006, from www.dest.gov.au/sectors/school_education/publications_resources/profiles/benchmarking_curricula

Donnelly will continue to argue for developing a national mathematics syllabus. Such a curriculum, he believes, would "leave no doubt about what needs to be taught at each year level, and what children are expected to understand and be able to do." Teachers can experiment with sequencing, he writes, but their main task should be "to devote their energies to improving teaching and hence the quality of student learning" (pp. 86, 87).

Still, adopting a national curriculum may not result in the improvements in achievement that might be expected, caution some educators. "The cultural activity of teaching—the ways in which the teacher and students interact about the subject—can be more powerful than the curriculum materials teachers use," write James Stigler and James Hiebert in "Improving Mathematics Teaching," in *Educational Leadership* (2004, p.16).

The authors analyzed videotaped mathematics lessons from a random sample of 100 8th grade classes from seven countries: the United States, Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, and Switzerland. Their study revealed a striking "homogeneity of teaching methods observed within each country" and "striking differences in methods" across other countries. In Japan, for example, students "spent an average of 15 minutes working on each mathematics problem during the lesson, in part because students often were asked to develop their own solution procedures for problems they had not seen before," write Stigler and Hiebert (p. 14). These types of "making connections problems" were typical in the classrooms of "high scoring TIMSS countries," the authors note. In the United States, however, teachers would most often transform a making connections problem into a "using procedures problem," giving students the formula and telling them "to simply plug in the relevant values" (p. 15).

Helping teachers teach better—by giving them opportunities to view the TIMSS videos, for example—should be the emphasis of reform, Stigler and Hiebert suggest. They note that "teachers who want to improve their implementation of making connections problems" may face a huge challenge: "They might never have seen what it looks like to implement these problems effectively" (p. 16).

Securing Support

Whether implementing a national curriculum, an NCTM *Standards*-based approach, or a more traditional program, most educators agree that the one thing they can all expect is resistance:

- **From scholars.** In one camp, researchers assert that traditional programs, which they believe are still emphasized in a majority of U.S. schools, focus too much on rote memorization and computation—students are given an algorithm for solving a certain type of problem and practice using that formula, for example. In the other camp, researchers assert that the emphasis on child-centered learning has gone too far. For instance, students may be asked to arrive at a formula for solving a problem—it's not given to them—and that's fine. But that only works, these researchers argue, if students have been given a foundation of skills with which and on which to build.
- **From teachers.** Some teachers embrace the student-centered, active learning aspects of the standards-based curriculum. They believe their roles as coaches in the classroom are appropriate and effective. Other teachers, conversely, have been successful teaching in a traditional program and see no reason to change. They believe they need to impart their content knowledge directly and efficiently, without the chaos typically associated with more hands-on learning activities.
- **From parents.** Some parents are excited when their children bring home stories of group projects or assignments that show how mathematics is used in real-world settings. Other parents want the subject taught the way it was taught to them. These parents are concerned that the new approaches are weak on computation.

Overcoming resistance to education reform is something school leaders should plan on, says Carol Burris, who is a principal in a district that implemented acceleration at the middle school level. As school leaders, "we should be prepared to explain why we do what we do, and show how our kids [benefit] in a very real way," she says. In her district, the number of students successfully completing higher-level math courses "zoomed up" after acceleration was implemented—a fact that, of course, school leaders shared with parents. “When you present people with facts, it becomes very hard for them to justify [their views] with only opinion,” Burris states. “Everybody has an opinion. I want to see the data. I want to see the facts. We have a lot of really good ‘show-me’ data that makes our practice hard to dispute.”

Although wooing parents to support school programs is an age-old practice, using data to win that endorsement is relatively new, say experts. New, too, is a greater awareness that communication about reform has to be two-way. “Few researchers have examined what students and parents make of the reforms, including how they might choose between *Standards*-based and traditional instruction,” writes Sarah Thuele Lubienski in “Traditional or Standards-Based Mathematics? The Choices of Students and Parents in One District” (2004, p. 340). Indeed, she observes that educators involved in reform often think that making a better case for standards will convince parents of their importance. Educators also often fail to ask students what they think about the changes occurring in mathematics classrooms—changes that call “for new roles for students” (p. 342).

Lubienski suggests that school leaders take time to assess and better understand the perspectives of those who resist reform, such as parents, students, community members, and some teachers. In doing so, she writes, they will be in a better position “to address community concerns and to implement mathematics instruction that best meets students’ needs” (p. 342) and may avoid some of the public outcry she found in her research.

A Study of One District

The opportunity to survey parent and student attitudes toward mathematics reform drew Lubienski to a Midwestern district of 5,000 students; it’s a mostly affluent, predominantly white student body, with Asian students making up most of the 16 percent minority population in the schools. In the mid-1990s, the district began changing over to *Standards*-based curricula funded by the National Science Foundation (NSF), implementing the *Investigations* program in the elementary school, and

Defining Exemplary Curricula

As inspiring as it is for teachers to see one of their colleagues effectively teaching a wonderful, *Standards*-based lesson, it can also be daunting: How can they possible create a similar lesson on their own?

"Fortunately, today's mathematics teachers are not expected to always create innovative units on their own as they may take advantage of the many exemplary instructional materials informed by the NCTM *Standards* that have been produced in recent years," write Raffaella Borasi and Judith Fonzi in *Foundations: Professional Development That Supports School Mathematics Reform* (2002, Becoming Familiar with Exemplary Instructional Materials and Resources, para. 1).

To be considered exemplary by the National Science Foundation (NSF), "a unit or comprehensive curriculum must be consistent with the NCTM *Standards*, designed by groups of specialists in mathematical content and pedagogy, and revised based on field tests in various instructional settings," write Borasi and Fonzi (Becoming Familiar with Exemplary Instructional Materials and Resources, para. 4).

Such materials usually include suggestions for planning lessons and orchestrating class discussions, examples of student work, tools and rubrics for assessment,

NSF-Funded Curricula

There are 12 *Standards*-based curricula that have been developed with support from the National Science Foundation (NSF). Four are middle school mathematics curricula and five are high school curricula:

Middle Grades Curricula

- Connected Mathematics 2
- Mathematics in Context 4
- MathScape: Seeing and Thinking Mathematically
- MATH Thematics

High School Curricula

- Contemporary Mathematics in Context
- Interactive Mathematics Program
- MATH Connections: A Secondary Mathematics Core Curriculum
- Mathematics: Modeling Our World
- SIMMS Integrated Mathematics

For full descriptions of each of these programs, go to the K–12 Mathematics Curriculum Center at www2.edc.org/mcc. You'll find a link to the recently published eighth edition of the *Curriculum Summaries.*

and opportunities for teachers to learn more about the mathematical concepts to be taught, Borasi and Fonzi state.

The authors urge school leaders and teachers not to feel pressured to create their own innovative lessons and units—"although, certainly, there's value in that." Adopt a coherent, exemplary curriculum instead, they advise. That way, educators will ensure "that students engage in a well-constructed sequence of worthwhile mathematics experiences" while teachers can "focus their energy on improving their instructional practices and evaluating their students' learning," write Borasi and Fonzi (Becoming Familiar with Exemplary Instructional Materials and Resources, para. 6).

piloting the *Mathematics in Context* (*MiC*) curriculum in grades 5 through 8 (*see p. 27 for a list of NSF-funded curricula*).

Despite the strong support of many stakeholders, including most of the district's math teachers, "heated controversy emerged among parents, school board members, and a few teachers about the transition to *MiC*," writes Lubienski (2004, p. 343). The district attempted to address parents' concerns, which were primarily about a drop in computation scores on the state standardized test. Still, "despite the complete rebound in computation scores"—and the fact that students performed well in mathematics concepts and problem solving—"dissension about *MiC* has persisted," she writes (p. 343).

The parental protest delayed the district's plans to implement a *Standards*-based program at the high school, writes Lubieski. Then, in the fall of 2000, the district introduced a four-year, integrated mathematics sequence. This time, to ward off dissension, district leaders opted to give parents, students, and teachers a choice between the traditional sequence (Algebra, Geometry, Algebra II, Pre-Calculus) and the *Standards*-based *Core Plus* sequence that integrates Algebra, Geometry, Pre-Calculus, and Statistics. Accelerated middle school students who began high school mathematics in 7th or 8th grade were also allowed to choose. "The district's mathematics cabinet—and many mathematics teachers and administrators—supported the integrated option and hoped that many students would choose it," writes Lubienski. "However, relatively few students did. Despite the district's efforts to promote the Integrated sequence, [fewer] than 18 percent of the 600 eligible students enrolled in the integrated option" (p. 349).

Why did so many students and their families prefer a traditional mathematics program? One factor was prior high-achievement in math: A greater percentage of the accelerated middle school students (27 percent) than high school students (13 percent) chose Integrated Mathematics. Lubienski attributes this outcome to the fact that the program was more actively promoted at the parent meeting for accelerated middle school students.

Another factor was the level of the parents' education. Children of highly educated parents were more likely to choose the integrated option, Lubienski found. "Whereas only 8 percent of students with noncollege-educated parents were in integrated mathematics, 31 percent of students with a Ph.D.-holding parent were in integrated," she writes (p. 350).

Parents and students provided many reasons for their mathematics choices, including

- **Learning style issues.** Several parents and students who chose traditional algebra talked about the instruction they preferred—primarily that they didn't want to work in groups (which the *Standards*-based curricula would require).
- **Concern about scope and sequence.** Several students said they chose traditional algebra "because they thought it would be confusing to learn mathematics in an integrated way" (p. 355). These students preferred to focus on algebra one year, geometry the next, and so on. One student explained, "I wouldn't like all of those mixed things and ideas; I like building up" (p. 355).
- **Questions about instructional philosophy.** Many parents didn't like the middle school *Standards*-based program and didn't want that approach to continue for their children in high school. Many parents and students viewed traditional algebra as "real" high school mathematics that has stood the test of time. "One student wrote, 'Algebra is real math. I can use Algebra later in life. It has been used for years and has been

successful. My dad is a scientist/mathematician, and he said integrated was worthless' " (p. 357).

Informing Reform

As Lubienski points out, this district's story "challenges the notion that students and parents will be 'won over' after several years of *Standards*-based instruction" (2004, p. 361). The study, she writes, "highlights the need to gain broad-based support for reforms, but also reveals the difficulty of doing so, given the diversity and deep-rooted nature of parents' and students' beliefs about secondary mathematics instruction" (p. 362).

Fortunately for reform-minded educators, however, the study also suggests strategies school and district leaders can use to answer some of the real concerns parents and students have about *Standards*-based math. School and district leaders need to

• Actively address the perception that algebra is "real" college-preparatory mathematics. "Provide clear information about ways in which local colleges are handling *Standards*-based mathematics credit," Lubienski advises (p. 362).

• Hold more parent or family math nights for a broader group of parents. Although parents of students in accelerated mathematics courses attended parent nights and discussed options with teachers, other parents, including those with fewer years of education and lower incomes, "were more likely to access information through school newsletters than parent nights," so their concerns couldn't be directly addressed (p. 362).

• Acknowledge learning style issues. It's important that students and parents understand, early on, the role of the student and the teacher in *Standards*-based classrooms, writes Lubienski. "Issues related to learning through problem solving, including the frustration that can accompany this, should be directly addressed" (p. 364). This includes explaining why some amount of frustration tends to be a necessary part of genuine mathematical activity.

• Help teachers become skilled in implementing *Standards*-based approaches "in ways that help students and their parents understand and realize the benefits of such approaches" (p. 364).

Using Technology to Enhance the Curriculum

Lubienski's study revealed why it's important to help students and parents understand the rationale for curricular choices they make. Many parents believe *Standards*-based mathematics shortchange computation skills, for example. NCTM's Cathy Seeley reminds stakeholders that computation is still "an important part of a balanced math program," but students need so much more than that. "Even in an era of calculators, we still want kids to do basic arithmetic." But more important, says Seeley, students "have to know what to do with the arithmetic they learn." They have to develop skills in geometry, data analysis, and algebraic thinking—and they can hone their computation skills "in the context of all that," Seeley states. "It's not necessary to limit the curriculum in order to develop computation—*even in an era of calculators*."

Once a subject of hot debate, most reform-minded educators today embrace using calculators and other technology to help students attain a deeper understanding of mathematical concepts. If technology isn't used appropriately, however, well-intentioned educators can unintentionally limit the curriculum in other ways.

Four common behavior patterns "handicap the potential of computers to promote higher-order thinking," writes Mary Burns in "Tools for the Mind" (2005/2006, p. 49).

One is that professional development is often focused on training teachers in skills rather than on how to use computers to enhance student learning. This diverts "needed attention from helping teachers understand the instructional practices best suited to capitalize on technology's potential, serving instead to hide or exacerbate weaknesses in instruction, lesson design, and assessment," writes Burns (p. 49).

It's critical that teachers have a good sense of the material they're presenting to students, agrees Joseph Siddiqui, dean of students and director of studies at the Main School of Science and Mathematics. That understanding includes how to use technology to help students grasp the content in upper-level courses, like AP Calculus.

A second behavior that can limit technology's effectiveness in the classroom is related to resources, writes Burns. Many districts fail "to provide such supports as long-term professional development in

Creating a High-Quality Mathematics Program

In 2002, Patrick Bathras was the first recipient of ASCD's Outstanding Young Educator Award (OYEA). He was honored for his innovative teaching strategies and strong commitment to student achievement.

Bathras, now principal at Severn Middle School in Arnold, Maryland, continues to actively promote a student-centered learning environment that he hopes will instill in students a positive attitude toward life-long learning.

Bathras recently shared his current thinking on high-quality mathematics and high quality teaching—all in service, of course, of improved student learning.

When you won the OYEA award, you were a mathematics teacher at a middle school in the Baltimore area. You had a passion for making math interesting and relevant to your students. Describe some of the lessons that were effective with your students.

Believe it or not, when I was young, I was one of those students who didn't like mathematics. When I became an elementary school teacher, however, I found that I loved to teach the subject. I never forgot my initial response to mathematics, though, and that's why I tried to make learning mathematics enjoyable.

One way I did this was to create a mathematical walking tour of Oriole Park at Camden Yards in Baltimore, Maryland. The walking tour enabled students and their parents to see mathematical concepts at play in the real world: they saw geometry in architecture, basic skills used in money handling, and how sports statistics are used. Many on the tour had never made such mathematical connections before. I distributed a math packet during the tour, which included information showing how the day's activities reinforced the math skills students needed to master in order to be successful on the state assessments. This was a fun way to experience, learn, and apply math concepts.

Why is it so important to help students see that mathematics can be interesting, hands-on, and fun?

Too many students are turned off to math—they say that it's boring, or too hard, or that they'll never use it anyway. Research has shown, though, that without a solid foundation in mathematics, these students' educational and career options will be limited. As a result, we can't let them just blow it off. We need to find ways to reach students, to show them math is in the world all around them, and that it's fun—something that shouldn't intimidate them. The stakes are just too high to not do so.

In Middle School Mathematics, an online course to which you contributed articles that had a practitioner's perspective, you outlined three components of a high-quality mathematics program. What are those components?

I emphasized problem solving, making math fun, and teacher expertise as three essential components of such a mathematics program. Why these three things?

1. Problem solving is the key to developing a deep understanding of mathematical concepts. Students have to have basic computational skills, but that just isn't enough in today's world. Students must be competent problem solvers because mathematical concepts and problem-solving situations coexist in everyday life.

2. It's important to make math fun for the reasons we discussed earlier—we need to turn kids on to math. Too many students consider mathematics to be their least favorite subject, and too many students do not pursue mathematics-related careers.

3. Teacher expertise is key. A high-quality mathematics teacher who is well-versed in teaching and learning strategies is important for the success of all students. A study conducted by retired University of Tennessee professor William L. Sanders showed that students who had three consecutive years of effective teachers in mathematics scored at the 83rd percentile in state assessments, whereas students who had three consecutive years of ineffective teachers scored at the 29th percentile—this is proof that effective teachers do make a difference children's lives.

In addition to being an educator, you're a father and husband. Your two boys are just starting their educational careers. What kinds of learning experiences do you and your wife, who was also an educator, hope for your children?

When you send your children off to school each day, you hope that the staff at the school will care for them as you would, allowing opportunities for emotional, intellectual, social, and physical growth. I hope that the school my sons attend promotes student growth and achievement, instilling a positive attitude toward lifelong learning. I also hope that the learning experiences are meaningful, effective, purposeful, and enduring.

And, when you ask your child, "How was your day?" it's always wonderful to hear: "Great—we had fun today!" Fun and learning should go hand-in-hand. When school is "fun," children want to recreate those experiences or yearn for similar ones, sparking their interest and thirst for learning.

We live in such a fast-paced, ever-changing, diverse world and the demands of society continue to mount. I expect my child's education to be a training ground in preparation for the real world.

Mathematics is all around us and it is a content area that is so important to a child's education. I want my own children to be prepared and proficient in mathematics, especially as it applies to real-life applications and their future career choice. Mathematics instruction isn't just for "math majors" or for those students that have an affinity toward mathematics, it's for everyone. Mathematics teachers can help to create classrooms that are conducive to all learners, regardless of student content area preferences. All students can truly succeed, and we owe it to our students—our own children—to ensure ways in which they can meet with success.

Source: Adapted from *Middle School Mathematics*, an ASCD PD Online Course, by Diane Jackson. Take a sample lesson at pdonline.ascd.org.

technology integration; access to sufficient hardware and software; creation of sufficient instructional time for inquiry-based, technology-integrated activities; on-site technical support; and instructional leadership to help teachers understand how they can use computers to extend and deepen student learning," she writes (p. 49).

Deepening student learning is what the use of technology is ideally supposed to achieve. Still, writes Burns, "in all the excitement about new ways of teaching with technology, we educators may have neglected to pose the most fundamental question: Are students really learning?" (p. 49).

Look to the student for clues about the effectiveness of teaching, advises John Bakelaar, assistant principal at Whitten Middle School in Jackson, Mississippi. When he taught middle school mathematics, he loved watching his students' faces when they could see a formula as a graph—thanks to the graphing calculator, which Whitten hooked up to an overhead projector so all students could see. "When you put a simple function on a calculator—$y = x + 2$—and the kids could see the line actually develop—their smiles are the reward," he says.

Bakelaar believes teachers have to use technology because students do. "It's today," he states. "If we sit in a classroom and say 'x-squared plus x-squared equals . . . ' [students] aren't going to listen," Bakelaar contends. Especially when some of those same students will come to class with examples of problems they've worked out on their own or with a picture of a car they drew on their calculator the night before—to which Bakelaar will always ask, "So, how did you do that?" Indeed, Bakelaar loves it when students use their calculators much in the same way they use other technology for entertainment. "They're at home playing with calculators, working on math," he says. "How wonderful is that?"

If the technology use leads to higher-level thinking, then Burns does think it's wonderful. Too often, however, educators "classify all software applications as cognitively and instructionally equal." This fourth common behavior pattern results in teachers frequently choosing "conceptually easy kinds of software—lower-order applications that, although engaging, focus on simple cognitive tasks—at the expense of more conceptually difficult kinds of software—higher-order applications that are more aligned with higher-order skills," Burns writes (p. 49).

The applications teachers most commonly use, for example, are what Burns calls "show-and-tell" programs that allow students to present information in different ways but don't really help them practice using their analytical and critical thinking skills. Using spreadsheets and databases, however, would provide that kind of learning experience, she states.

"Spreadsheets demand both abstract and concrete reasoning and involve students in the mathematical logic of calculations. They enable learners to model complex and rich real-world phenomena. Students practice their critical thinking skills by making assumptions, coding assumptions as variables, manipulating variables, analyzing outcomes, and evaluating and displaying data both quantitatively and visually," writes Burns (p. 50).

Databases also help to cultivate higher-order thinking skills. "By its very taxonomical nature, database design can help students systematically organize, arrange, and classify data according to established criteria," Burns writes. "Such activities require students to think inductively (in aggregating data) and deductively (in disaggregating information).

Yet databases, like spreadsheets, are 'difficult,'" she notes. "So students rarely use them for analytic purposes" (p. 50).

To break the bad habits of technology use, Burns suggests teachers remember that technology such as calculators and computers are "mind tools" that should be used to boost critical thinking and allow for learner-centered instruction.

"Teachers are often reluctant to use technology," says Wesley Brown, a mathematics resource teacher in the Prince George's County (Maryland) School District. Teachers may eschew technological tools for philosophical reasons, researchers point out, or because they don't really understand how to use it in their classrooms. Better training, says Brown, can show just how effective using technology, like calculators, can be. "Take a lesson on slope. There are examples in the book that discuss what slope represents," Brown explains. But teachers need to see how a calculator can easily diagram what happens when there is change in the *y* value over the *x* value. "Technology is particularly useful when teaching students that concept," he states.

"I've been very fortunate that I had mentors who used calculators," says Jeffrey Chaffee, a math curriculum enhancement teacher at Remington Middle School in Franklin, Massachusetts. When he taught 8th grade math, Chaffee went to many workshops that showed how to use graphing calculators effectively—and frequently. "If you don't use it, you'll lose it," he notes.

Chaffee is now dedicated to helping teachers in his district learn to use the device. "I want to make sure people don't avoid using it," he says. "It's such a good tool." He notes that this is particularly true in 8th grade, when students learn the basics of going from an equation to the graph table. "If you then want to use real-life data, making a graph by hand would be a nightmare," he says. Drawing a graph by hand is also time-consuming—time that would be better spent analyzing the problem, says Chaffee.

Reflections ◆ ◆ ◆

• **Is it time for a national curriculum?** Although calls for a national curriculum are not new, proponents of the idea make compelling arguments. In 1997, for example, Richard M. Haynes and Donald M. Chalker suggested that for a nation to have "world class schools," they must have a national curriculum. Such a curriculum would simplify textbook publishing, help all students perform better on national standardized tests, and better meet the needs of students in a mobile society, they write in "World-Class Schools: What does it mean to be 'world class'? And how close are U.S. schools to getting there?" (Haynes & Chalker, 1997). Implementing a national learning program for school mathematics is an idea worthy of discussion.

• **Support for educational programs requires two-way communication.** It's not easy to implement a *Standards*-based program when confronted with the "I learned it (or taught it) this way, and I turned out all right" argument. As Lubienski points out, educators must seek out the perspectives of all stakeholders—including students—to truly understand resistance to change. And, as Burris points out, it's helpful to show them the data.

• **Technology can enhance the curriculum—when used effectively.** Teachers need to become proficient users of new technologies such as graphing calculators, spreadsheets, and other programs. These can profoundly influence how effectively certain mathematical topics are taught in school.

3 Bringing the Curriculum to Life in the Classroom

The whole is more than the sum of its parts.

—Aristotle (circa 330 BCE)

Whether for better or for worse, the United States does not have a national curriculum. In many other industrialized nations—the Netherlands, Finland, and South Korea, for example—the question of what students should learn is answered at a federal level. In the United States, determining what basic education each child should receive is a state-level—or sometimes district-level—responsibility. Each of these approaches to curriculum development has its advantages and disadvantages, but no matter how it's created, the effectiveness of any curriculum is determined by its implementation in the classroom.

Using Standards As a Framework

When he talks about standards, William Schmidt, distinguished professor from Michigan State University, is referring to "a well-defined set of things that students should know, grade-by-grade and within schools." Schmidt thinks that standards should be "nationally determined" so curriculum materials, like textbooks, could address shared learning objectives and serve as better tools for teachers.

Still, says Schmidt, curricular materials should "never be confused with curriculum. The particulars about what materials to use should be the purview of teachers and school districts," he says, so long as they are used to "implement a specific vision for math."

Standards are "the *what* of what you'll teach," says Robin Fogarty, an internationally known professional development consultant. In a 2001 ASCD conference presentation, Fogarty advised teachers to use standards to create kid-friendly activities that will entice students to learn. Fogarty proposed that the only way to do this is by grouping learning objectives and outcomes together "so that content, process, and performance standards are combined and clustered, textured and tiered, and infused and integrated." In doing so, she says, educators create "a dynamic curriculum."

As Fogarty suggests, teachers can be creative in designing lessons that help students meet the standards. Here's a sampling of some imaginative approaches:

- A grade 10 geometry class addresses several state standards in basic math, geometry, and statistics through a necktie project. Students design the ties and then create an accompanying advertisement. The tie design must reflect the students' understanding of geometric and spatial properties; the advertising copy must show the students' understanding of marketing statistics and the ability to make a persuasive argument.
- Middle school students use a real recipe and a set of measuring cups and spoons to learn about equivalent fractions. Students are asked to use as few utensils as possible to measure the ingredients. As a result, students discover they can use the ¼ cup measuring cup six times to get 1½ cups of flour.
- Grade 8 students participate in an international ozone monitoring project. Students collect their data locally then share their findings via the Internet with students at other schools around the world. Students also graph and analyze the data. Because they care about air quality and are collecting real data for a real purpose, students take their work seriously.

Innovative Lessons

Using imagination and innovation in lesson design is the norm for the teachers we interviewed for this book. These teachers, most of whom are winners of the 2003 Presidential Award for Excellence in Mathematics and Science Teaching, understand the importance of helping students find connections between the math they study and the world in which they live. In this chapter, these innovative teachers describe how their lessons help students "make sense" of mathematics and bring the standards-based curriculum to life in their classrooms.

The lessons we'll share in this chapter address many of the standards described in the Nation Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics*. We'll identify which of the five content area and process standards each lesson was designed to target. What becomes obvious, however, is that these rich

Lesson: Fill 'Er Up

Lesson Description: Working in teams of three or four, students will mediate a dispute at a soda factory. To do so, students must determine if the current policy used to determine who will receive worker bonuses is fair. Students must support their conclusions with appropriate data. This will require that students

- Identify the factors that affect the rate at which a container is filled with a substance.
- Consider how a container's shape affects the fill rate.
- Create graphs that visually support proposals made to the client about the bonus policy.

As part of a series using two-dimensional objects, this lesson asks students to examine the properties of geometric shapes and use those properties for problem solving. Students will apply appropriate measurement techniques and use their understanding of patterns, relations, functions, and spatial visualizations to analyze change and to interpret phenomena in real-world contexts.

Prerequisites: Prior to this lesson, students learned to graph in the Cartesian plane and to look for trends and correlations in data. They have learned to measure length in customary and metric units, and they have studied patterns—numerical, geometric, and statistical—and learned to describe them using various representations, including writing rules, using algebraic expressions and equations, and drawing tables and graphs.

Content Standards Addressed	Process Standards Addressed
☑ Number and Operations	☑ Problem Solving
☑ Algebra	☐ Reasoning and Proof
☑ Geometry	☑ Communication
☐ Measurement	☑ Connections
☐ Data Analysis and Probability	☑ Representation

activities address many more—if not all—of the standards for secondary mathematics.

Kristen Scanlon has always enjoyed mathematics. "I love the problem solving, the logical sequencing of things, and understanding the number system," she says. Scanlon was teaching 7th grade pre-algebra at McNair Middle School in Fayetteville, Arkansas, when she was nominated for the 2003 Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST). For the competition, she submitted a lesson that exemplified one of her goals for her students: to help them see that patterns and data are not just "artificial math concepts." Her students, she writes in her PAEMST application, "would always ask 'When will I ever use patterns and graphing in REAL life?'" At an NCTM institute that focused on geometric reasoning, Scanlon found a lesson that would answer that question and appeal to the "naturally curious and energetic nature" of her middle school students—a hands-on activity that supported "their emerging reasoning skills as they move from the concrete to the abstract."

The Lesson: Synopsis

The lesson begins with a brainstorming session. Students work with the teacher to generate a list of factors that might affect the rate at which soda bottles are filled on an assembly line. Scanlon observes that students will typically think of such factors as the width of the container's opening, the thickness of the substance being poured, and the type of machines used to fill the containers. Once all ideas are shared and discussed and logged in the students' math journals, the students learn what their task will be:

> You are part of an engineering team [that] has been hired by the Fill 'Er Up Bottling Company to solve a dispute among the workers. The company bases the workers' bonuses on how many bottles they fill each day. The workers think that the machines are filling the bottles at different rates, which makes the bonus criteria unfair. You've been hired to research this

problem and recommend to the company how best to solve it. (Scanlon, 2003, p. 4)

The research phase of the problem helps students develop the concept of capacity, writes Scanlon. She divides her students into teams of three or four and gives each team a bottle, a small cup, and a container of colored water. Each team has a different shaped bottle—some are cylindrical, some are wide at the bottom and become narrow at the top, and still others have a reverse hour-glass shape: narrow at the bottom and top, but wide in the middle. Students begin by estimating how many cups it will take to fill their bottles. Then they use the small cups to fill the bottles with water and measure the height of the liquid in centimeters after each cup is added, graphing their results as they proceed.

Scanlon's job during this lesson is to observe, responding to students' observations and questions. "I notice several things," Scanlon writes. "Students with the cylinder-shaped bottles comment that the liquid is rising the same amount each time; groups are developing their own measuring techniques, such as using two rulers to make a more accurate measure; students are unsure what type of graph they should use, and I remind them that line graphs show change over time—a concept they are still struggling to understand."

The students then post their graphs on the wall for the entire class to consider. Scanlon asks them to decide which bottles go with which graphs (*see Assessment, p. 45*). "The students are mostly able to match the graphs with the bottles, and their discussion helps them develop a deeper understanding of the concepts," Scanlon writes. Students compare their ideas to those of their classmates and they make convincing arguments to support their opinions. "One student even changed his mind about a bottle based on the arguments of a fellow student," writes Scanlon.

The Lesson: Reflections

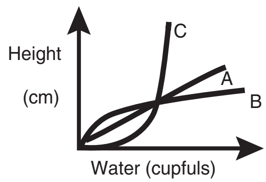
Scanlon was pleased with several aspects of the lesson. She writes in her PAEMST application that the students were "interested in solving the problem from the beginning." The lesson, she writes, provided different entry points for students of all ability levels, and, because "students used physical models to deepen their understanding of measurement and capacity concepts, all student experienced success with these activities—even students who had been previously tentative or hesitant. The hands-on nature of the activity made that possible." Students were also able to demonstrate their understanding in a variety of ways—by drawing pictures, making graphs, presenting their results orally, and writing explanations.

In addition to the students' success with the lesson, Scanlon observes that the activity also gave her a chance to use the skills she had honed during her years of teaching. "I have learned to ask good questions and then guide students toward discovering the answers," Scanlon writes. "Good questions are ones that don't have a simple answer: they encourage students to infer, predict, hypothesize, synthesize, and look back at their prior knowledge." The lesson exemplifies the best of her teaching, Scanlon writes, because it shows "the fruits of my efforts to provide an intellectually and physically safe environment for student learning, one where expectations are set high and clearly conveyed to the students. . . . Students show throughout this lesson that they buy into the activity and take responsibility for their own understandings of the important math concepts. They are developing into autonomous mathematicians."

And what about those bonuses? Students in Scanlon's class recommended that the management of the Fill 'Er Up Bottling Company should use different criteria to determine bonuses. Start, the students advised, by ensuring that, regardless of shape, all the bottles had the same capacity. They added that counting the number of bottles filled in one day isn't really fair and that the fill rate should be used instead. Compare how many ounces per hour are deposited into the bottles, students suggested.

"They're very astute," Scanlon observes.

ASSESSMENT

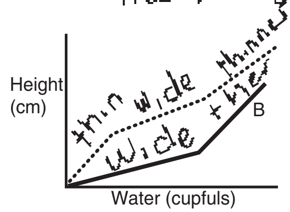


1. The graph above represent the data from three different bottles

a. Which bottle is the tallest?
Why? C, because the line goes higher

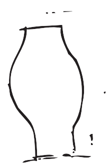
b. Which bottle is the widest?
Why? B because it is the shortest and takes more water

c. Which bottle holds the most water?
Why? B because the line is longer

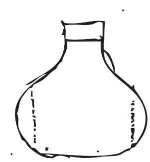


2. For the diagram above, design a bottle which could produce graph A and graph B

BOTTLE A



BOTTLE B

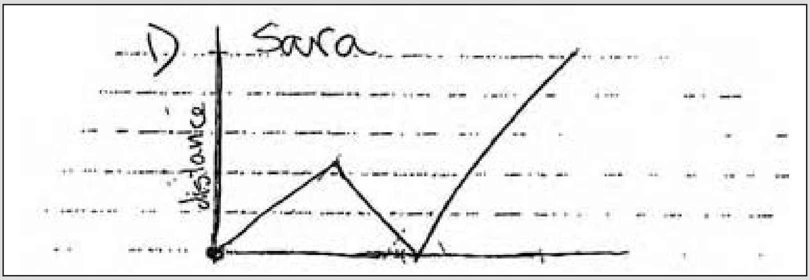


According to Scanlon, the assessment above shows that this student has internalized the key concepts and uses those ideas to answer the questions.

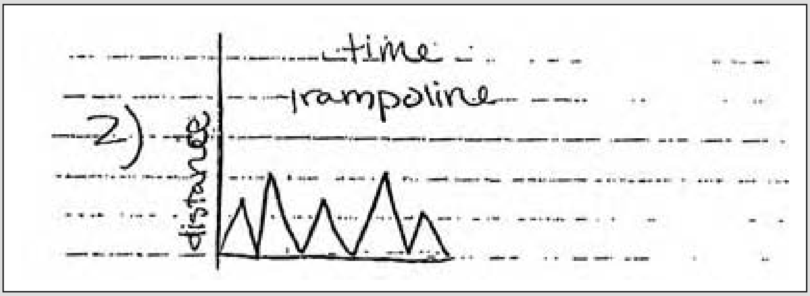
Drawing Graphs

Students were asked to graph time on the horizontal axis and distance from home on the vertical axis for each of the following scenarios:

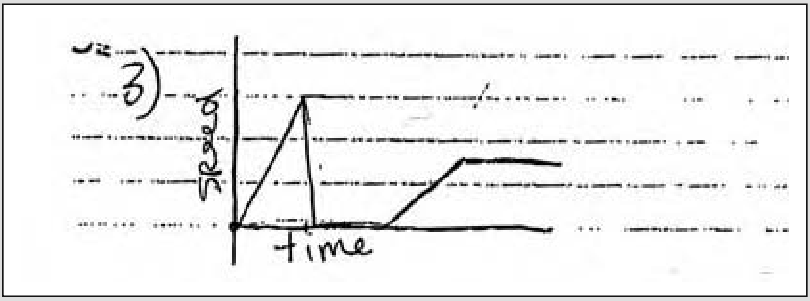
1. Sara walks from her home to the store. Halfway to the store, she realizes that she forgot to bring money, so she turns around, returns home, gets her money, and then walks all the way to the store.



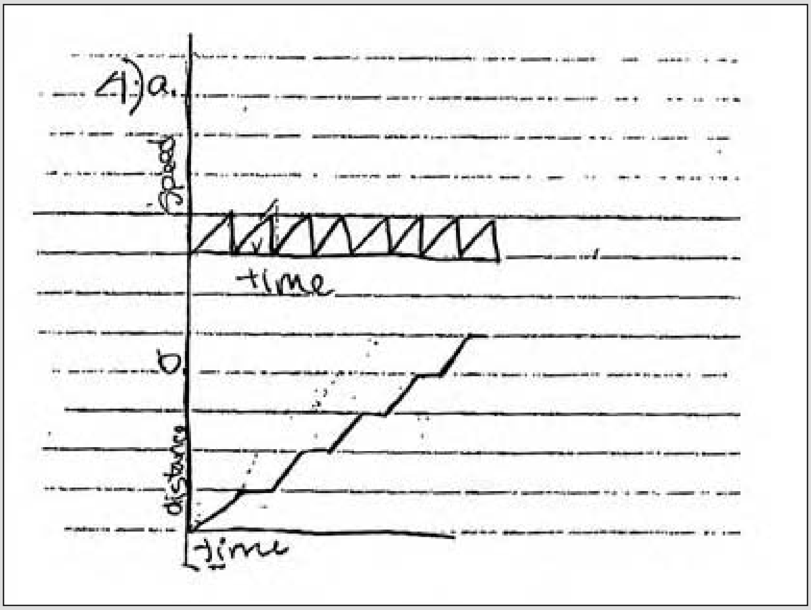
2. Rashid is jumping on a trampoline.



3. Kendra is speeding along the highway and is stopped by a police officer. The officer gives her a ticket, then she continues on her way.



4. Carlos lives in a large city and travels to school on a local bus that stops every block to let passengers on and off. Graph the time and speed of the bus, as well the time it takes Carlos to get to school.



These graphs provide instructional feedback for Scanlon. The graphs the student drew "are basically correct," Scanlon writes. "They go up when they should go up and down when they should go down, but there are a few details [that] should have been included that are not. The graph for question 2, for instance, does not account for the child being two or so feet off the ground before starting to jump on the trampoline. Also, a trampoline jumper goes up and then comes back down lower than he was before he started jumping—and that should be on her graph.

"The fourth Graph illustrates the bus accelerating and decelerating, but it does not include time for the stops.

"The student has clearly mastered some of the targeted concepts but will benefit from more practice relating graphs to everyday occurrences."

Lesson: Radius Square and *Pi*

Lesson Description: Students are given a circle and four different colors of squares with a side length that is equal to the radius—the radius square. Students must then determine how many radius squares fit into the circle.

Prerequisites: Prior to this pre-algebra lesson, students will have spent five or six years learning about nonstandard units of length, weight, and capacity. They will have explored concepts of two- and three-dimensional geometry. They will have defined and labeled the following parts of a circle: center, radius, diameter, and chord, and they will have explored the meaning of circumference of a circle. Students will have also used appropriate tools to measure area, perimeter, circumference, radius, and diameter in the standard (English and metric) systems. They will have, for example, computed circumference of various circles and verified results by measuring with string.

Content Standards Addressed	Process Standards Addressed
☑ Number and Operations	☑ Problem Solving
☑ Algebra	☐ Reasoning and Proof
☑ Geometry	☑ Communication
☐ Measurement	☑ Connections
☐ Data Analysis and Probability	☑ Representation

If there is one mindset John Bakelaar wants his students to adapt, it's recognizing that math "is not just a class you have to take—[it] is a survival skill." When he was a teacher at Peeples Middle School in Jackson, Mississippi, Bakelaar would regularly introduce scenarios that illustrated how real people use their math skills to make better decisions: "You want a part-time job at WalMart and learn that your hourly wage is $5.55." Is taking the job worth it? Bakelaar asks students, reminding them to also remember that their take-home pay is not the same as their gross pay. How many hours a week must someone work to make the job worthwhile? he asks.

In addition to incorporating real-life uses of mathematics in his lessons, Bakelaar was also determined to create "more hands-on activities," which, even at the secondary level, make it easier for students to grasp key concepts, he says. Take ratios, for example. Rather than risking the blank stares that may result when a lesson starts with: "Today we're going to talk about ratios," Bakelaar suggests that teachers give students a packet of M&Ms and ask, "How many yellow to blue M&Ms do you have? How many red to green?" A student may answer that he has three blue and seven yellow. "Now, if I just said 3:7, some student may grasp that I mean three parts of seven, but I don't think they'll understand it as clearly as when they can see it right in front of them and they can explain it to you," Bakelaar says.

The Lesson: Synopsis

Bakelaar's lesson, Radius Square and *Pi*, reflects his instructional philosophy. The objective: to help students arrive at their own definition of *pi* before they are introduced to the formal definition.

Begin Bakelaar says, by giving each student four different colors of radius squares and a circle with the radius equal to the side length of the square.

Then, have students place the first square entirely inside the circle. Have them cut the second square and place it in the circle without overlapping the first square. Make sure students use all the second square before moving on to the third square.

Continue with the third and fourth squares in the same manner.

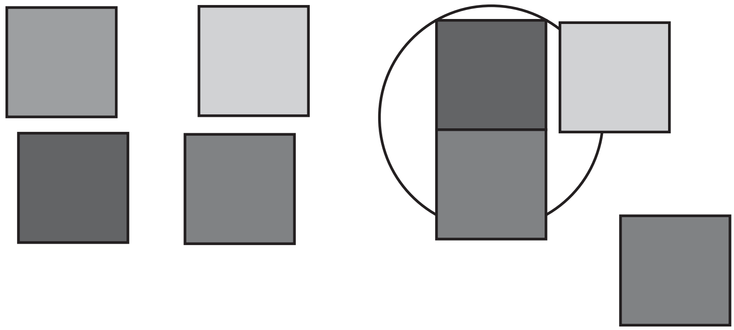
Save the remainder of the fourth square, divided into smaller squares, to estimate how much of the fourth square students were able to fit inside the circle (*see Radius Square and Pi example, p. 50*).

"Students will come to see that they can fit three squares completely inside the circle and a little of the fourth one," writes Bakelaar in a PowerPoint presentation on the lesson. "The radius square fits 'three and a little more' into the circle. *Pi*!" (2001, p. 7).

The Lesson: Reflections

Bakelaar, who is now assistant principal at Whitten Middle School in Jackson, Mississippi, says students like this lesson because they ultimately

Radius Square and *Pi*



will see what *pi* looks like. Using a manipulative makes it easier for students to estimate, Bakelaar says. "It's very interesting to see their initial estimations."

In the class discussion that followed the hands-on activity, students—all of whom had some familiarity with *pi*—became quite excited as they translated "three and a little bit more" into its decimal equivalent.

"I asked students, 'What is that number close to?'" After a short pause, one student piped up, "Why, that's *pi*, Mr. Bakelaar! Is *pi* exact?" Through the activity, students came to realize that *pi* is an estimation, Bakelaar explains. Students see that there is more than one possible answer, he says; they see that "my 3.15 and your 3.21 are pretty close," and that no one student is going to have the same product.

The Radius Square and *Pi* lesson is engaging, Bakelaar notes, which addresses a question he often received from other teachers (Bakelaar was also the math coach at Peeples Middle School): How can I get my students to stay on task? "When I've shared this lesson or lessons similar to this, other teachers will often say, 'Wow, it's a great lesson. I didn't have any problem with my students.'" That's to be expected, according to Bakelaar. "If students are involved and engaged," classroom management just isn't an issue, he says.

Lesson: Tessellation T-Shirt

Carrie Chiappetta has a hard time imagining how she might teach mathematics in isolation from other subjects. The 6th grade teacher at Scofield Magnet Middle School in Stamford, Connecticut, works with six other teachers on her middle school team to create interdisciplinary lessons that weave together math, science, technology, language arts, social studies, art, and reading. "We want students to see how all these

Lesson Description: This lesson occurs during the unit on geometry. Throughout the year, students are given a Problem of the Week (POW). This particular problem requires that students make a tessellation that will then be put on a T-shirt. Almost an entire class period is dedicated to the discussion of this assignment.

Students follow the steps on the assignment sheet and check them off as they are completed. For the written portions of the POW, some students may elect to make a poster to answers all the questions while others may opt to write a more formal report; either way is acceptable.

To complete the POW, students need to perform a number of tasks. First, students need to rewrite what they are being asked to do in their own words; that way, both the student and the teacher know that the problem is understood.

Next, students consider what the problem requires them to do and determine what materials they may need to solve it. Students need to identify if this problem is similar to those they have previously completed and what strategies they could possibly use.

Prerequisites: Prior to this pre-algebra lesson, students were able to identify, classify, draw, and describe two-dimensional geometric shapes. They also understood the spatial relationships of shapes including symmetry, geometric reflections, rotations, and translations and developed an understanding of congruent and similar figures. In addition, students had a background in discovering and developing patterns.

A familiarity of the POW format was also necessary for this project. Because students had been doing POWs over the course of the year, they were comfortable with the set-up of the POWs and, therefore, knew what was considered an appropriate and complete answer.

Content Standards Addressed	Process Standards Addressed
☐ Number and Operations	☑ Problem Solving
☑ Algebra	☑ Reasoning and Proof
☑ Geometry	☐ Communication
☑ Measurement	☑ Connections
☐ Data Analysis and Probability	☑ Representation

[subjects] fit together in the world—it makes math more meaningful for them," Chiappetta says.

Because the school's focus is centered on math, science, and technology, and because Chiappetta and her team "are allowed to make our own curriculum" based on state and national standards (including NCTM's), students at Scofield have had opportunities to explore the natural world. "We are an inner city school, so our kids love it when they have a chance to go out into the woods and ponds."

One year, Chiappetta's students conducted a leaf transect, for example. Students were given a 100-meter section of property. Every three meters, they were to stop and pick up three leaves. The students kept track of the types of leaves they collected on a graphing calculator. Students then used the data to create stem and leaf plots; determine, on average, what kinds of leaves abound in the area they covered; and draw to scale samples of the leaves collected. This project, which addresses science, math, social studies, and technology standards, gave students a first-hand look "at what our [school] property looks like," says Chiappetta.

The Lesson: Synopsis

The T-shirt tessellation lesson helps illustrate how math can inform art. Chiappetta introduces the concept by first showing examples of works from artists like M. C. Escher (see www.cromp.com/tess/people/TessHome.shtml for other artists' tessellations). "I don't call the pictures 'tessellations,'" writes Chiappetta in her PAEMST application. Instead, "we talk about and brainstorm the characteristics that all the pictures have in common. We then make a list of these characteristics, and from this list students are then able to give *me* the definition of the word," she explains.

Once her students have created a definition, Chiappetta arranges for them to "experience the production of a tessellation." She gives them pattern blocks with which they make their own repeating patterns. When their tessellations are complete, students inspect others' designs. "We then talk about which design seems to be a tessellation, according to the definition that they discovered earlier," Chiappetta writes.

Next, Chiappetta and her students make a tessellation of a fish together. "I model the construction of the template step-by-step and the students imitate. We then talk about how the pieces that were cut off need to be put back on the shape and cannot be thrown out," she writes. She then demonstrates different ways to make it tessellate on the overhead and students begin this process on paper in class and finish it for homework.

This is an important juncture in the lesson because Chiappetta uses the assignment to determine which students understand tessellation and which students need to review the concepts. "Students who are able to make a tessellation out of the pattern blocks and who come next period with the fish tessellating on the page and with the paper colored in a pattern are then ready to begin designing a template of their own. I am then able to spend time with students who had difficulty, while the others are designing and testing their own template," she writes.

Once students have made and tested their template, they are ready to trace it on their T-shirt. To do so, students put a piece of cardboard inside their shirt, secure it with a rubber band, and trace the template onto the front of their T-shirt with a permanent marker. Students can then color their tessellation with fabric paint.

Students aren't finished, however. Because nurturing the ability to communicate mathematical concepts is a key process standard, Chiappetta's students must explain how they made their T-shirt template. Students must define the shapes they used, describe why they cut the pieces in the manner they did, and explain the positions and orientations of the pieces they used in their tessellations. Students also need to discuss what other shapes work for tessellating, writes Chiappetta, and they need to give examples of where they see tessellations *outside* math class. "All of this would assist the students in answering the question, What do tessellations have to do with math?"

The Lesson: Reflections

The T-shirt tessellation project "is excellent for a wide range of abilities," Chiappetta states in her PAEMST application. "Throughout the years that I have done this project, I have found that the students who excel in arithmetic sometimes have a difficulty with the spatial visualization. . . . I have also found the opposite: students who do not excel in arithmetic find the spatial visualization . . . easy," she writes.

Chiappetta likes the lesson because it provides time in class to meet the diverse needs of students: while some students are designing their own template, she can work with students who need more assistance.

"I thought that this would be a creative and innovative way to have students discover, investigate, and produce a tessellation and its connection with mathematics and the real world," Chiappetta writes. "Instead of giving students the definition of a tessellation, they were able to come up with it on their own; instead of having students color tessellations from a book, students are able to design one of their choice and see the different shapes and the transformations of shapes; and instead of having students draw it on a piece of paper, which could be lost easily, they put it on something that they may keep just a little bit longer—a T-shirt.

Lesson: Standard Deviation

"In a lot of ways my philosophy [of teaching] is traditional," says Joseph Siddiqui, dean of students and director of studies at the Maine School of Science and Mathematics. "I have a lecture style. . . . I want students to really internalize the material as we go . . . and [my] kids aren't in cooperative learning groups," he explains. Siddiqui also describes himself as traditional when it comes to assessment practices: daily homework, pop quizzes, four tests a year, and a final.

Still, students in Siddiqui's classes don't just take notes, take a test, and move on. Instead, Siddiqui designs lessons (such as the one he describes in the PAEMST application) that require students to discover mathematical concepts on their own. And his students work in groups outside of class to complete three "math majors" each semester—six a year. These large problems take students through several steps to discover the mathematical concept (*see examples, p. 56*). The math majors "involve all the material we're covering, but students really have to apply it," says Siddiqui. The math majors "build students' ability to communicate mathematically" and help students better grasp the information covered in class.

Siddiqui's instructional approach is appropriate for his clientele: he teaches at a boarding school to which students apply and, although some "really struggle in math and science—they're not all geniuses"—the

Lesson Description: Standard deviation is a key statistic used in analyzing data, but it's a concept that is often difficult for students to understand. This lesson, therefore, first demonstrates the need for such a statistic. After showing that no previously studied measures of center/spread are able to adequately differentiate between four very different sets of data, students immediately realize a new statistic is necessary and work together to determine what that new statistic should be.

Throughout the lesson, students are never given a definition for standard deviation. Instead, they are asked to determine which of the four given lists has the most spread. As a result of this lesson (which includes a great deal of class discussion and student hypotheses), students are able to determine the formula for standard deviation on their own.

The benefit is that this lesson dispels the mystery behind standard deviation. Students develop their own understanding of the need, concept, and formula for this statistic and, as a result, are able to better describe an entire set of data and are better prepared to continue their studies of statistics.

Prerequisites: Prior to this lesson, students gained an understanding of the three measures of central tendency (mean, median, and mode). During the first half of a 90-minute block, students studied boxplots and the five-number summary (minimum, first quartile, median, third quartile, and maximum). This material led into a discussion of the first two measures of spread (range and interquartile range).

When students were introduced to range and interquartile range, they quickly determined that these first two measures did not amply describe an entire set of data. As a result, they knew a new statistic was needed. Because of experiences in science labs, students had familiarity with the notion that distances as measured from the mean were important. Discussion of the measures of central tendency made this notion understandable, though the exact definition was still a mystery.

Content Standards Addressed	Process Standards Addressed
☐ Number and Operations	☑ Problem Solving
☐ Algebra	☐ Reasoning and Proof
☑ Geometry	☑ Communication
☐ Measurement	☑ Connections
☐ Data Analysis and Probability	☑ Representation

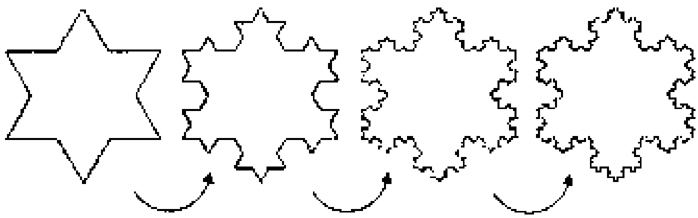
Before Calculus BC **Name: ______________________**

DUE by 12:25 pm on Friday, January 31, 2003 - NO EXCUSES!!!!!
Get started Early – Though wonderful, this could be a tough one!

Solutions to all major problems must be detailed and clear. They must include **ALL of your work**, they must contain **complete sentences** and they must be easily readable to educated people in ANY field. Your actual write-up is **AT LEAST** as important as your final solution. **See your "Guide to the Majors" to review all other expectations!**

1. A Most Wonderful Fractal **(15 points)**

The Koch Snowflake is constructed by trisecting the sides of an equilateral triangle, and replacing the middle section with a new (smaller) equilateral triangle. These triangles in turn have their sides trisected, and still smaller equilateral triangles replace their middle sections. This process is carried on infinitely many times. (See below.)



a) Show that as each set of smaller triangles is added, the perimeter of the figure (i.e. the length of the curve) is always given by a term in a geometric sequence. (Let each side of the original triangle equal 1 unit).

b) Show that the perimeter of the snowflake becomes infinite as the number of times new triangles are added increases.

c) What is the area inside the snowflake?

Source: Used with permission from J. Siddiqui, Maine School of Science and Mathematics, Limestone.

students are motivated and comfortable in a traditional setting that emphasizes rigor. To be effective teachers in such an environment, Siddiqui says he and his colleagues at the school must have "a really good sense of the material they're presenting—and of what will come later." Math is such a cumulative subject, he explains. "It's very difficult for a math teacher to provide a foundation [for students] . . . if they aren't somewhat familiar with what comes next."

The Lesson: Synopsis

Reinforcing students' foundational understanding of the three measures of central tendency (mean, median, and mode) and introducing the notions of the five-number summary, boxplots, and two measures of dispersion (range and interquartile range) fill the first part of a 90-minute block. After that discussion, Siddiqui moves on to standard deviation, which is "often a mystery because so many see the statistic as a magic number that results from a formula with no real sense of what the statistic represents," he writes in his 2003 PAEMST application. Siddiqui tells students that he wants them to gain an intuitive sense about what standard deviation represents and then derive a formula based on that understanding.

To start, Siddiqui writes four different sets of data on the board and asks students to calculate each of the three measures of center as well as the range and interquartile range for each. Despite clear differences in the sets, each mean, median, and range proved to be exactly the same. "Students realized none of these statistics were strong enough to denote differences in the sets," Siddiqui writes. "The mode and interquartile range proved different for each set, yet students realized these statistics were not necessarily strong enough to illustrate other differences that might occur because of their narrow focus on only certain values in the sets." Students grasp, rather quickly, that they will need a new statistic to capture the differences between the complete sets of data.

What does your intuition tell you about the spread of each data set and how they relate to each other? Siddiqui asks his students. "A debate ensued over which sets truly were most and least spread out," he writes.

In order to settle the debate—and to make the sets more concrete—Siddiqui lines his desk with *Star Wars* figures. He tells students to imagine that a message needs to be transferred, without error, from one of the characters to each of the others. What placement of characters would most easily allow for this to happen? Siddiqui asked his students. Students agree that it would be easier to pass the message along when the figures' placement results in the least spread; transference would be more difficult if the figures' arrangement resulted in the most spread. "This open dialogue regarding concrete figures led students to the essential idea of measuring spread as compared to the mean and enabled students to accurately determine which sets of data had the greatest and least spreads," Siddiqui writes. "Students were then able to understand how to determine spread by considering each element of the set."

The class then focuses on the four numerical sets on the board and determines which sets were most and least spread out. The ordering of the spread of the two other sets was less obvious, and "thus we turned our attention to the subtleties of spread. For example, if one data point is very far from the mean and one is very close to the mean, is that any different than having two data points that are 'just right' from the mean? It is indeed very different and that difference is essential in understanding the definition of standard deviation," Siddiqui writes.

Once students secured an intuitive sense of standard deviation, they finished ordering the four data sets according to what they believe their standard deviations to be. "We then derived the formula for calculating standard deviation based completely upon our own understandings of the statistic."

Students are then shown how to use a graphing calculator to determine numerical values of standard deviations for each set. "We then quickly and easily confirmed our intuitions were correct," writes Siddiqui.

The Lesson: Reflections

Overall, Siddiqui was pleased with the way the lesson progressed. "The student involvement and discussion throughout was pivotal and gave a heightened sense of ownership and understanding, while providing me opportunities to correct misunderstanding," he writes. Using *Star*

Wars figures lined across the desk helped show students what they needed to consider when determining the definition of standard deviation.

According to Siddiqui, the highlight occurred when the students themselves were able to dictate the formula for standard deviation, based on their own intuitive understandings. "Using the calculator to confirm our analysis further solidified student understandings," he writes.

Lesson: Introduction to Quadratic Equations

Lesson Description: In this 7th and 8th grade Algebra I class, students will use the TI-83 graphing calculator to discover the effect that changing *a*, *h*, and *k* in the equation $y = a(x - h)^2 + k$ has on a parabola. The patterns they discover will allow them to move from any equation in this form to its respective graph.

Students will learn that *a* controls the width and direction of the parabola, *h* is the x-coordinate of the vertex, and *k* is the y-coordinate of the vertex.

Prerequisites: Although students will have had no previous experience with this material, they will have done extensive work with the graphing calculator. When they see the equation, $y = 1(x - 0)^2$, students will know that the relationship is not linear or exponential.

Content Standards Addressed	Process Standards Addressed
☐ Number and Operations	☑ Problem Solving
☑ Algebra	☐ Reasoning and Proof
☐ Geometry	☑ Communication
☐ Measurement	☐ Connections
☐ Data Analysis and Probability	☐ Representation

Students may never have to solve a quadratic equation in the work they will ultimately do, says Jeffrey Chaffee, the math curriculum enhancement teacher at Remington Middle School in Franklin, Massachusetts. But it's what happens in the process of solving that equation that will prove invaluable to students' academic and professional success. "It's the mindset," says Chaffee, "the understanding of how to get from point *a* to point *b*, the ability to be persistent and to seek help when need be."

It's also being willing to use the tools of the trade that make it easier to focus on the important aspects of any job. That's why Chaffee doesn't shy from incorporating the use of graphing calculators in his lessons. By using a calculator, students can better spend their time analyzing the problem and discussing "what if" questions: What would happen if we change these numbers? What would happen if we tried it this way? Using technology "encourages those kinds of questions," says Chaffee. "And that's where innovation comes in."

The Lesson: Synopsis

In the introductory part of the lesson, Chaffee spends time reviewing new vocabulary words with his students—such as *quadratic equations* and *parabola* (he would introduce *vertex* later in the lesson)—and shows examples of each. "I referred back to their work with linear relationships to give them a means of comparison," writes Chaffee in his 2003 PAEMST application. "I reminded them how changing the slope and y-intercept in a linear equation affects the graph of a line and told them we are going to discover something similar with quadratic equations and parabolas."

Chaffee then displays a graph of $y = 1(x - 0)^2 + 0$ and asks students to compare all other parabolas to the one displayed. He then distributes and reviews the direction sheet with students, explaining that their objective is to find the roles of a, h, and k in the equation $y = a(x - h)^2 + k$.

Chaffee divides the students into groups of three or four and for the next 15 minutes, students work on the problem, changing one variable at a time. Students make hypotheses, test predictions, and record their results. Meanwhile, Chaffee circulates "from group to group asking questions to find out what they have discovered. I [pose] questions to help them verify and refine their conclusions."

At the end of about 15 minutes, Chaffee asks students to share their discoveries, which gives him an opportunity to address any misconceptions students might have. Through questions and examples, Chaffee leads the class to the conclusion that h and k are the coordinates of the vertex.

"I called on different students to present their group's findings. We started by focusing on the effect that changing a had on the parabola," Chaffee writes. Students ultimately decide that *a* determines the width and direction of the parabola. They have more difficulty explaining the roles of *h* and *k*, Chaffee observes. "Eventually, I [led] them to see that *h* is the *x*-coordinate of the vertex and *k* is the *y*-coordinate of the vertex," he writes.

During the conclusion of the lesson, Chaffee shows students how they could make a fairly accurate sketch of a parabola by using information from their equation. He asked students to consider the equation: $y = -3(x - -2)^2 + 4$. Does the parabola open up or down? he asks students. (The answer: down.) Is the opening of the $y = -3(x - -2)^2 + 4$ parabola narrower or wider than the $y = 1(x - 0)^2 + 0$ parabola? (The answer: narrower.) What are the vertex coordinates of $y = -3(x - -2)^2 + 4$? (The answer: -2, 4.) Chaffee then puts up a new equation and asks students to try and answer the same questions, comparing the new parabola to the $y = 1(x - 0)^2 + 0$ parabola.

The Lesson: Reflections

"Many parts of this lesson went even better than I had anticipated," writes Chaffee, who felt the lesson was well paced—not too fast or too slow. "Students were engaged, productive, and enthusiastic about the process. They asked each other great questions that pushed their partners to back up their statements." While students worked, Chaffee was able to meet with many students individually or in their groups; by the end of those first 15 minutes, he had a good sense of which students understood the concepts and which didn't.

"The class summary segment was also quite successful," Chaffee observes. "Students presented their ideas about each variable, and we kept refining them as a class until we had concise, mathematically accurate statements."

Chaffee was especially pleased that students had some time at the end of class to practice what they had learned. "Sometimes when the material is difficult, we spend all class discovering the concept and don't get to apply it until the next day," he notes.

Lesson: Using Everyday Situations to Explore the Pythagorean Theorem

Lesson Description: This lesson will help students recognize and apply mathematics in contexts outside of math. What is more removed from the math classroom than a baseball game or your teacher having been accidentally locked out of her house at 6 a.m.?

Students will take a mathematical concept—in this case, the Pythagorean theorem—and apply it to real-life situations. By seeing the right triangle in everyday situations, the students will clearly experience mathematics outside typical confines.

Prerequisites: Prior to this lesson, students had limited experience with, or knowledge of, the Pythagorean theorem. In previous lessons, the students learned how to use the calculator. The majority of them had heard that $a^2 + b^2 = c^2$ but did not know how to apply this formula or what it was used for.

Content Standards Addressed	Process Standards Addressed
☐ Number and Operations	☑ Problem Solving
☐ Algebra	☐ Reasoning and Proof
☑ Geometry	☑ Communication
☐ Measurement	☑ Connections
☐ Data Analysis and Probability	☑ Representation

Nancy Foote is a teacher-on-assignment in Higley, Arizona—a booming suburb of Phoenix. “In 2001, we had 350 students here,” says Foote. “Today, the student population is around 7,000.” Such growth has, naturally, put a bit of a strain on the local school system, which has been hiring teachers with diverse backgrounds—some are experienced, some are brand new, and some come with deep content knowledge but little teaching experience.

Enter Foote. After close to 20 years of teaching math to middle and high school students, she now teaches math to teachers who need ideas on ways to effectively deliver the curriculum. She begins by looking at the learning goals for the quarter and brainstorming different ways to teach those lessons.

Take slope, for example. “I try to make sure teachers have a conceptual understanding of slope,” Foote says. She often uses children’s stories and other books as teaching aids. “Take the tortoise and hare—you can tell who is going faster by the shape of the line,” she explains. Then, she asks teachers, "Where do you see slope in your work? Where do kids see it in their world?"

Looking for that connection to students' lives is important, Foote states. "Take a real-world situation and mathematize it," she urges. That way, math is not forced. "It's more natural."

The Lesson: Synopsis

Foote was teaching at an alternative high school, working with a diverse group of students as young as 13 and as old as 19, when she was nominated for a presidential award. The lesson about the Pythagorean theorem that Foote submitted for the competition reflects her commitment to helping students make connections between the math they learn in school and their own worlds.

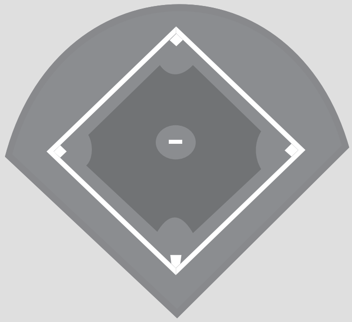
She introduces the lesson by telling the students about a time when she was locked out of her house. The only way to get in was through a second-floor window, 25 feet off the ground. If she leaned the ladder against the house on an angle, 10 feet from the house, how long would the ladder need to be to reach her window?

Before tackling her problem, Foote and her students spend time exploring geometric properties that are key to understanding the Pythagorean theorem, such as defining right angles and right triangles, defining a, b, c, and *hypotenuse*; squaring a number; adding a^2 and b^2; finding the square root of that sum (which equals c^2) to determine c; practicing the mechanics of the process, and evaluating real-world problems.

In videotaping her lesson, for example, Foote explains that she needs a microphone cable that will allow her to cross, diagonally, to the corner of the room. If the room is 20 feet by 30 feet, how long of a cable does she need? Students compute their answers on individual whiteboards, holding them up so Foote can check their work. "I use whiteboards to make constant and frequent assessments of students' learning and understanding," Foote writes in her 2003 PAEMST application. What's more, the whiteboards helps keep kinesthetic learners engaged, she notes. Students found that they needed a little over 36 feet, but that they couldn't round off to 36 because then they wouldn't have enough cable to reach the corner. "You can almost see the light bulb go on in

Take Me Out to the Ballgame

"Have you ever noticed that a baseball diamond is actually a square turned sideways? Do you see the two right triangles?" Nancy Foote asked her students. "And, did you know that the distance between each base is 90 feet?"



So why, she asked, is it harder for the catcher to throw a runner out at second base than it is at first or third?

The answer, of course, can be found by applying the Pythagorean theorem, she explained. If the catcher is standing at home plate, how many feet does he have to throw the ball to reach second base?

Note: The answer is on p. 65.

their heads when they realize that close is good but sometimes isn't good enough," writes Foote.

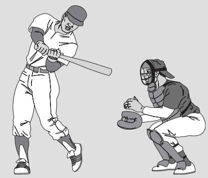
After a few more real-world examples (*see above*), Foote asks students to work in groups to complete an in-class, hands-on activity that allows the students to apply the theory on their own. She has used her classroom whiteboard and the floor to create right angles, placing marks at various heights on the whiteboard and on the floor; each mark on the board measures so many feet high, such as 3, and students determine that this is a; each floor tile equals 1 foot, so they measure the number of tiles they must cross to reach their mark to find b. How much ribbon would be needed if they were to stretch it from the mark on the whiteboard to the mark on the floor, thus finding c?

Foote circulates the room as the students work, offering assistance and checking for understanding. When one group arrived at a solution rather quickly, Foote discovered that one student had figured out the answer and was eager to try his solution, but two members of his group had "no idea what he had done." Foote then gave the group an additional problem, asking the two reticent students to take the lead. This encouraged them to ask questions to obtain the information they needed to solve the problem, Foote states. It also reinforced the idea of community: students must work together to help each other in grasping the concept, she says.

To conclude the lesson, Foote asks students to look for right triangles in their daily lives; they will share those examples in class on a later day.

Take Me Out to the Ballgame

Answer: The catcher has to throw the ball a little over 127 feet.



The Lesson: Reflections

One of her strengths as a teacher, Foote writes, is her "ability to take abstract concepts and apply them to daily life." The lesson on the Pythagorean theorem is a good example of this, she says. By showing how the formula can be used to help her find a ladder long enough to reach her second-floor window, Foote illustrates how a seemingly remote mathematical principle is actually a real-world tool that they can use to solve problems they could easily encounter. These moments, when students see math at work, are most gratifying to Foote.

Lesson: The Biorhythm Task

In 1989, Don Karlgaard attended a National Science Foundation geometry workshop that forever changed his approach to teaching. Karlgaard recalls returning to his classroom "convinced that I should discard my textbook and start the upcoming school year with concrete learning experiences instead of abstract proofs."

That same year, as a result, Karlgaard decided to introduce his students to constructions and geoboards before moving into coordinate geometry. His students also spent time in the computer lab, using geometry software to explore and to discover ideas about triangles and other polygons, he writes in his PAEMST application. "I became a learning

Lesson Description: This project is an assessment that allows students to apply what they learned during their study of trigonometry transformations. The assessment replaces the unit "book" test with authentic activities: students must develop three equations that model each student's physical, emotional, and intellectual cycle.

Prerequisites: Prior to this project, students worked together in groups of two or three to look at periodic phenomena like heights of a roller coaster, swings of a pendulum, current flows, rotating gears, hours of daylight, temperatures, and orbits. Students also used an equation for a sine wave to determine various properties of the wave based on scale changes and translations. Students used real-world data to determine the equation for a sine wave that fits the given data. In the unit preceding the biorhythm project, students should have learned the basic facts of trigonometric equations, understood scale changes and translations, and learned to model periodic data.

Content Standards Addressed	Process Standards Addressed
☐ Number and Operations	☑ Problem Solving
☐ Algebra	☑ Reasoning and Proof
☑ Geometry	☑ Communication
☐ Measurement	☑ Connections
☐ Data Analysis and Probability	☑ Representation

helper and not a teller. I could not believe how enjoyable my geometry classes became."

Karlgaard's approach to teaching has evolved during his long career—he's spent 34 years as a 9–12 mathematics teacher and now teaches at Brainerd High School in Minnesota. What hasn't changed is his deep regard for his students. "The students are my clients," he says. Karlgaard tries to get to know all his students "and understand their needs and work with them." This attitude influences his classroom environment, he suggests. "You won't see rows of desks in my classroom—I have tables," he says. "Almost every day, students will work in groups."

The ultimate goal, says Karlgaard, is for his students to know mathematics and to communicate with other people as they try to solve math problems. He writes, "I tell my students frequently, 'We are all friends here, trying to help each other learn mathematics.' When a student asks for help with a problem, I usually reply, 'What did your partners say when you asked them?' I work hard at developing dynamic small groups who, in turn, work together cooperatively and effectively."

The Biorhythm Task

Product

• A trigonometric equation for each pattern.

• A spreadsheet showing the values for all three biorhythmic patterns for the last month and the next two months.

• A graph corresponding to the spreadsheet.

• Communication of last month's experiences along with expectations for the next two months.

Description of Task

At the end of the 19th century, a German physician named Wilhehn Fliess wanted to establish a mathematical relationship between an individual's date of birth and the date of one's death or illnesses. Others have since expanded and adjusted the original idea, and the concept is now known as "biorhythm."

According to this idea, there are three biological patterns that begin at birth and affect disposition during the course of a lifetime. These patterns do not determine what will happen on a particular day, only how the individual is likely to feel as events occur. There is a physical cycle with a 23-day period, an emotional cycle with a 28-day period, and an intellectual cycle with a 33-day period.

On your day of birth, each pattern starts at zero and begins to rise in a positive phase, during which the energies and abilities associated with the cycle are high. Then, gradually declining, the patterns cross the zero point midway through their complete periods and continue into a negative phase in which capabilities are low. Then, increasing amounts of energy are picked up as the negative phase turns upward until, at the end of the pattern, the zero point is re-crossed into the positive phase, and the whole cycle begins again.

Your assignment is to develop a mathematical model to illustrate this idea, using your own birth date. Develop trigonometric equations to model all three patterns. Create a spreadsheet to calculate values for graphing the cycles for last month and the next two months. Use the graph to communicate relationships between the cycles and your recollection of good and bad experiences last month. Also use the graph to predict high and low times for the upcoming two months.

The Lesson: Synopsis

Karlgaard's instructional philosophy is evident in how he views assessment. An assessment activity is not much different from a learning activity—students should learn while doing assessments, Karlgaard says. His submission to the Presidential Award competition, therefore, was actually an assessment he wrote in the early 1990s to address the Minnesota math standards. ("Those standards were closely aligned with the NCTM standards," he says.)

"Accurate measurement of what students are learning and the ability to perform with content and processes are what all good teachers hope to discover in evaluation of their assessment," writes Karlgaard. "Can students apply the skills and facts that they have learned to create a project or to solve a problem?"

The Biorhythm Task (*see p. 67*) asks students to use the facts and skills they have learned about trigonometry transformations to develop three equations that model each student's physical, emotional, and intellectual cycles. "Students show that the number of days they have been alive is the phase shift; the 23, 28, and 33 day biorhythm cycles are the periods for the three different equations. Changing the cycles to a value to be inserted into an abstract trigonometric equation demonstrates procedural knowledge."

Karlgaard likes this assessment because "students are very interested in finding out something new about themselves. Since each student usually has a unique birth date, each student's equations and biorhythms will be different."

Karlgaard didn't explain what biorhythms are, opting instead to have students research and interpret all three cycles as a homework assignment. This gave the students ownership, he writes. Students "became active participants in the class discussion about biorhythms instead of me just telling the students what they needed to know about the subject."

The Lesson: Reflections

"I have noticed quite a difference in the quantity and quality of student's writing in this assessment," Karlgaard writes. In previous years, although his students "did a great job on the pure mathematics of the Biorhythm Project," students' performance in the written communication area was inconsistent. Karlgaard then realized that he needed to supplement the curriculum with more writing instruction.

His students, Karlgaard notes, "feel good about what they have learned." Each year, the results of the assessment "have told me that my students are learning the basics and, more importantly, can apply what they have learned to a new context."

Many high school teachers have suggested to Karlgaard that the Biorhythm Task is "too tough" for their students. Karlgaard doesn't buy that: "I have been very pleased with the quality of work my students have done in the Biorhythm Task," he states.

Lesson: Is Democracy Fair?

For Jason Cushner, one of the more rewarding aspects of traveling to Washington, D.C., as a Presidential Award winner was the opportunity to speak before Congress about the state of mathematics education in the United States. Cushner, one of four awardees chosen to provide testimony, talked about many things, including innovative math programs, how to retain good teachers, and the need for teachers to have more control in creating or choosing curriculum. One of his most important points, however—and one reflected in his teaching—is the need to make math accessible to more students. "I have made a career out of teaching math to those who believe they can't do math," Cushner told the committee.

He does this by giving students problems that they see "as useful and interesting. A good math problem has to help students understand

Lesson Description: The students organized and used the people per congressional district data to model representation in the House of Representatives and to solve the social problem of fair and just representation.

To complete the task, students had to interpret the initial data, find appropriate strategies to compare the data, and use this comparison to address the larger problem. They developed and tested a variety of conjectures to decide which method was the most fair.

The students used three communication and reflection steps: a presentation of their results to their group, a presentation to the class with a discussion, and a written paper as part of their portfolio explaining their methods and findings.

Students worked with data that was represented in various ways: through a real-life situation, a table, an equation, a scale model, and a graph. They enhanced their knowledge of different apportionment methods and fairness to choose a best method. This mathematical analysis was connected to social studies, how to set up a democracy, and how we represent the views of people in a large nation.

Prerequisites: This task requires that students know how to work well with others. Students in this class did a great deal of group work before this task.

This task also requires that students are comfortable presenting their results to a larger audience. Therefore, students are often in front of the class presenting their ideas or running discussions. Because students are responsible for and engaged in the learning process, each student is consistently working at his or her current level of understanding.

Content Standards Addressed	**Process Standards Addressed**
☐ Number and Operations	☑ Problem Solving
☐ Algebra	☑ Reasoning and Proof
☐ Geometry	☑ Communication
☐ Measurement	☑ Connections
☑ Data Analysis and Probability	☑ Representation

their world, while also being complex and challenging," he says. Cushner was a teacher at Eagle Rock School in Estes Park, Colorado, when he received his award. The lesson he submitted illustrates his ability to pick topics that engage students, particularly those who previously were not successful in math—or school. "When students overcome their math phobias, their whole persona changes," says Cushner, now a lead educator at the Watershed School in Boulder. Helping students find that they are academically capable "shifts their paradigm," he observes. Math achievement leads to school achievement: It gives students more confidence, Cushner asserts. "They know they can be scholars."

The Lesson: Synopsis

Most of the students in Cushner's class initially perceived school, and especially mathematics, as too difficult. Rather than trying to "break down the mathematics into simple steps," however, Cushner takes a somewhat counterintuitive approach: he gives his students complex, open-ended problems to solve. What would happen if students "didn't get" those simple steps? he asks. "It would reinforce any phobias they might have."

Cushner helps ensure that his students will realize success in solving "problems of some magnitude" by making sure his lessons account for differing levels of mathematical ability. For example, he describes the lesson he submitted for his award as having multiple points of entry that will engage students.

The lesson begins with a review of apportionment, giving students an opportunity to articulate the concept. "I then explain that we will use data to compare apportionment methods," Cushner writes in his 2003 PAEMST application. He emphasizes the social importance of this issue—Montana once sued the U.S. Congress over this issue—and introduces mathematical concepts that "might help us understand the repercussions if an unfair apportionment system existed," he writes.

Cushner then reframes the problem:

> You are a congressional committee in 1790 and need to decide on a fair apportionment system. This decision is crucial to the success of the democracy.

First, students set up a scale model of the representation to visualize and gain a basic understanding of the problem. Then, they compute the people-per-representative ratio. "These concrete tasks initially immerse students in the situation," writes Cushner.

As students work in groups to determine which apportionment method is the fairest, Cushner circulates, asking students "questions that will best develop conceptual understanding and problem-solving skills but will also allow students to maintain ownership of the process." In this part of the lesson, students make and test conjectures on what the data means and how to use it.

The class then meets to discuss their findings and come to a consensus that the Huntington-Hill method is the best for determining the number of representatives each state has in the U.S. Congress. "Because students have experiences in reaching agreement on solutions, it takes almost no framing on my part and they are able to question, challenge, and discuss each others' ideas. This is where we get everyone on the same page in making sense of the concepts and results," Cushner explains. "When the students agree the Huntington-Hill method is the best because it has the lowest numbers of people-per-representative and the least variance in those numbers, it is an ideal lead into the concept of range. They have, through their own process, virtually discovered a formal mathematical concept." Cushner then gives students the definition of range; students apply their data, watching as it concurs with what they had determined as the best method. They also analyze range for its strengths and weaknesses, setting students up for using future statistical tools.

The Lesson: Reflections

When given a real-life problem that connected math and democracy, students "showed both enthusiasm and persistence," writes Cushner. "I was pleased with the way they used different modalities to explore the data: representing ratios with beans on scale models, making calculations of the data in tables, writing equations, and graphing data on calculators."

The students, according to Cushner, were effective in articulating the mathematics concepts and explaining how they related to a fair democratic system. "Through their answers to my questions and the discussion at the end of class, they showed that they understood what the data meant," he notes.

Cushner also thought his questioning technique was effective in helping students gain a conceptual understanding of the data. When working with one group, for example, Cushner questioned a student on the meaning of the numbers. "It is important to me to gain a solid understanding of what a student thinks before I try to add new knowledge or processes," he writes. Therefore, he poses a series of questions designed to probe the student's level of understanding and to encourage the student to grapple with the concepts underlying the problem.

Cushner challenged students' mental models, and with the student in this case, he asked her if her vote would make more of a difference in Delaware or Virginia. "When she answers, I don't confirm her answer but ask her, 'How come?' I ask this to get her to think about the validity of her answer as opposed to being dependent on me as the 'expert,' " Cushner explains. He then brings another group member into the conversation and asks what the data would look like in an ideal situation. "This allows them to picture the ideal data spread so they can use that as a comparison to their own data. This line of questioning pushes them to think about the numbers and the implications of the data." For data to be analyzed accurately, students need to be strong at interpreting the meaning of the numbers, Cushner writes.

This lesson achieved the goal of having students use tools to interpret data and make inferences, Cushner says. Students were active and generating the information the majority of the time. "The strongest evidence that this lesson is effective is illustrated by students like Alley," writes Cushner. She came to Eagle Rock School "thinking she couldn't do mathematics." Now, however, she can formulate a sophisticated analysis of data and enjoy math, he says. She now believes—knows—that "she can do mathematics."

Assessing to Learn

Bringing the curriculum to life in the classroom requires that teachers use instructional strategies that emphasize real-world connections. That, in turn, helps ensure students' active engagement, as the teachers featured in this chapter have shown. These teachers also believe that assessment can be used as tool to guide the direction of their continued instruction. The emphasis is on helping students learn.

This philosophy is as old as education itself, as Managing Editor Marge Scherer observes in the November 2005 issue of *Educational Leadership*:

> Since Socrates first started asking questions, testing students has been part of teaching and learning. A teacher asked his student questions so that he knew how to guide his student and so that the student knew what else he needed to learn. (p. 9)

Scherer continues that, for just as long, "probably since the first school put more than one student in a classroom," teachers have been using tests to compare and rank their students (p. 9).

Does assessment today focus too much on the testing—and sorting—of students? Scherer asks her readers to consider that question and offers a variety of articles from assessment experts that insist the answer doesn't have to be "yes." These authors maintain that educators can focus on assessment for learning by using formative assessment in the classroom.

Formative assessment, writes Scherer,

- Blurs the line between instruction and assessment
- Involves students.
- Provides meaningful feedback.
- Examines how students think.
- Leverages large gains. (p. 9)

Of course, the innovative teachers included in this book provide some of the best examples of how to use assessment to enhance student learning.

Blurring the Line

"If someone were to walk into my classroom while students were working on this authentic assessment [the Biorhythm Task], one would not notice much of a change from how students were working together on previous days to learn the concepts necessary for this task," says Don Karlgaard.

Nor would anyone see much difference in Karlgaard's role in the classroom. He is "continuously assessing students' progress" by observing, questioning students' assumptions, and providing guidance. This blurring of the lines between instruction and assessment can help teachers improve their instructional practices, note assessment experts (Scherer, 2005, p. 9); assessment can help teachers refine their lessons.

"It's important to check for understanding throughout a lesson—to see if you are challenging all of the students," says Colleen Miller, a 6th grade math teacher at Northley Middle School in Aston, Pennsylvania. Taking cues from students means that she needs to stay flexible—and make immediate adjustments, if necessary, says Miller, who was featured in the ASCD video program *Using Classroom Assessment to Guide Daily Instruction: Techniques for Classroom Assessment*.

In the lesson videotaped for the program, for example, students use two calculators—a mathematical calculator and a scientific calculator—to determine why it's important to follow an order of operations. Does it matter whether the calculator adds or multiplies first? Miller asks. Through observation and questioning, she can see if the students understand the lesson and what modifications should be made, right then and there.

"If I see that the students had difficulty explaining why we need order of operations, I might do a follow-up lesson," she explains (Checkley, 2004). She uses an analogy and asks students to think about baking a cake. Why are ingredients added in a certain order? Students then have to explain the importance of order in that scenario—and then transfer that understanding to math.

Miller also uses a relationship-building strategy as a form of assessment. At the end of her class, students fill out an "exit card"—index cards on which they list what they learned from the lesson and what concepts they think need to be reinforced. These "reassure the students as they leave my room that [I know] they did a good job—they don't leave the room feeling frustrated [because they] did an excellent job but couldn't comment on it. In addition, they are reassuring me that they have grasped onto the concept of the lesson," she says. The cards also give Miller guidance in modifying her plans. They "help me see what I could do differently" (Checkley, 2004).

Involving Students

Nancy Foote uses whiteboards to frequently assess her students' learning. "My students are comfortable with whiteboarding, and they willingly hold up their boards for me to evaluate their level of understanding of a specific concept," she writes in her PAEMST application.

One technique Foote particularly likes is called Give Me Five. The in-class assessment requires students to answer the question, If you were going to take a five-question quiz, right now, on material exactly like this, how many would you get right? By having students hold up the appropriate number of fingers—zero through five—Foote sees immediately which of the students think they understand the material and which think they need additional instruction.

"When I compare this self-evaluation to the information I obtain from checking their whiteboards, I am able to determine if the child is aware of [her] own level of understanding," Foote writes. "Do they know that they are struggling with a concept? Or do they think they are doing okay?" This information is critical in helping Foote address misconceptions students may not be aware they have.

Meaningful Feedback

In all the examples of good teaching in this chapter, it's evident that the emphasis is on continuous learning and improvement. What teachers say to students, how students respond in turn—all reflect a desire to deepen understanding of mathematical concepts. The dialogue in the classrooms described reflects a respect for the students' intellect, and the teachers understand that, for feedback to be useful, it has to be specific. So, rather than merely saying, "good job," their comments guided students to consider new information or different perspectives

"General praise in no way helps me do my work any better," states Steven Levy, an expert on assessment featured in *Using Classroom Assessment to Guide Daily Instruction* (Checkley, 2004). "If you say, 'I really like that essay,' I have no idea what you're talking about. What part of the essay was it that you liked?"

Rubrics for Self-Assessment

In "Seven Practices for Effective Learning," Jay McTighe and Ken O'Connor write that "the most effective learners set personal learning goals, employ proven strategies, and self-assess their work" (2005, p. 16).

Rubrics, the authors write, can help students "become more effective at honest self-appraisal and production self-improvement" (p. 16). Take a look at the following rubric, for example:

Analytic Rubric for Graphic Display of Data

	Title	Labels	Accuracy	Neatness
3	The graph contains a title that clearly tells what the data show.	All parts of the graph (units of measurement, rows, etc.) are correctly labeled.	All data are accurately represented on the graph.	The graph is very neat and easy to read.
2	The graph contains a title that suggests what the data show.	Some parts of the graph are inaccurately labeled.	Data representation contains minor errors.	The graph is generally neat and readable.
1	The title does not reflect what the data show OR the title is missing.	The graph is incorrectly labeled OR labels are missing.	The data are inaccurately represented, contain major errors, OR are missing.	The graph is sloppy and difficult to read.

Comments: ____________________

Goals/Actions: ____________________

Source: From *The Understanding by Design Professional Development Workbook* (p. 183), by J. McTighe and G. Wiggins, 2004, Alexandria, VA: ASCD.

In using this tool, students can determine if they have met specific criteria; for example, how does the title of their graph describe the data displayed? How well is the data represented on the graph?

Students check the box on the left if they believe they have meet the criteria. The teacher then uses the square on the right side of the rubric for his evaluation. "Ideally, the two judgments should match. If not, the discrepancy raises an opportunity to discuss the criteria, expectations, and performance standards. Over time, teacher and student judgments tend to align," write McTighe and O'Connor (2005, p. 16).

The rubric also includes space for feedback and student goals and action steps, the authors note. "Consequently, the rubric moves from being simply an evaluation tool" to a "robust vehicle for feedback, self-assessment, and goal-setting" (McTighe & O'Connor, 2005, p. 16).

When teachers are specific, and feedback is used to further understanding or refine a final product, students then see how to "sit with their own work or with their peers' work" to determine what that work demonstrates," says Levy. Students begin to ask, What do I like about this work? What is there of quality? This begins a process of "naming the criteria that contribute to excellent work," he says (Checkley, 2004).

Once students understand what makes for excellent work, they can then be sources of feedback to each other, Levy suggests. This, he says, is "one of the most powerful techniques for really helping to improve work." Making the classroom a safe place for this to happen "is an important job" for teachers, he adds (Checkley, 2004).

Kristen Scanlon found this to be true. Her students would regularly present in class, and they came to expect that their ideas would be challenged. As a result, says Scanlon, her students had "become quite good at putting their thoughts into words."

This was evident during the Fill 'Er Up exercise (*see p. 43*), when students had to explain how a particular graph reflected the shape of a particular bottle. "Students would disagree with each other in a productive way, giving plausible reasons for why they disagree," Scanlon writes in her PAEMST application. Students received immediate feedback from each other that either validated or clarified their thinking about concepts, she says.

Looking at Student Thinking

One of the best ways to understand how students think is to ask them, says Jason Cushner. When he taught at Eagle Rock, he asked his students to create portfolios that provided evidence of their learning. The students, he writes in his PAEMST application, attached three to five pieces of evidence (homework and classwork) that best demonstrated their learning. “When students write their understanding, it gives me excellent insight into what they do and do not understand.”

The writing process helps students solidify and retain what they’ve learned, adds Cushner. At the end of the portfolio, students can test those skills on new problems to see if they can apply the skills to new concepts, he says.

Portfolios are often used because teachers, administrators, parents, and children become dissatisfied with standardized test scores that “don’t describe children well,” Beth Hebert observed at a 2000 ASCD Annual Conference session. Portfolios, said the principal of Crow Island School in Winnetka, Illinois, “show a depth of work” that such tests simply can’t (*Education Update*, 2000, p .7).

Portfolios help show how self-awareness can be built, for example. “If there is a curriculum for meta-cognition, it is portfolio,” Hebert noted. “The process of selecting and making decisions about how to organize a portfolio stimulates that kind of thinking in students” (p. 7).

Portfolios also contribute to teacher growth. Through portfolios, teachers find that students “can become competent participants in assessing their own learning,” Hebert said. Teachers then begin to ask: Is this portfolio about my teaching, supported by student evidence? Or, is this portfolio about student learning, supported by my assistance? That subtle shift in perspective is a first step in truly giving students ownership of their portfolios and their learning, she explained (p. 7).

Leveraging Gains

“Each teacher finds his or her own way of incorporating formative assessment into daily practice and school culture,” writes Scherer. And, as more educators understand what daily classroom assessment can tell them about student learning, “the practice of formative assessment becomes more powerful” (2005, p. 9). Powerful in boosting student achievement—and in helping teachers improve, assessment experts note.

"I think teachers need assessment in order to improve in their craft as much as students need it in theirs," says Levy in *Using Classroom Assessment to Guide Daily Instruction*. "And if I think about the process that students have to go through in order to achieve excellence in their work, I think it's very similar for teachers. That process always involves trying something out, seeing what it is that makes good teaching, and naming that together" (Checkley, 2004).

"I'm a better teacher because I've had time to reflect on my lessons and watched others teach," says Jeffrey Chaffee. "Teachers would do so much better if they were given time to bounce their ideas off somebody else," he says, adding that teachers need to feel free to say, "This lesson didn't work. Can you help me figure out why?"

Getting feedback from other teachers is a key to remaining effective in the classroom, agrees Don Karlgaard, who belongs to what he calls "a math best-practices group." Learning from his peers fuels Karlgaard's motivation. "I get fired up," he says. "The workshops give me another month's worth of energy to go on."

The group recently focused on assessment practices. Teachers worked on a problem in which a volleyball player takes 225-milligram pills every eight hours. Her body filters out a certain percentage. How much of the substance is still in her body after 10 days? The teachers "do the activity as though they were the high school students," Karlgaard explains. In doing so, the teachers have a better grasp of what the assessment is asking of students and gives them an idea of the different directions students might take the problem. Then, teachers can plan for some of these contingencies.

As Scherer points out, the sorting function of testing threatens to dominate in schools today. "With the number and kinds of standardized tests proliferating," she writes, "testing also serves a reporting function: Parents, policymakers, and teacher all look to tests as the definitive proof that students are learning" (2005, p. 9).

The proof of student learning, in reality, is in what happens every day in the classroom, many educators state. And fortunately, when done well, meaningful classroom assessment can lead to improved performance on standardized tests. "Research indicates that using assessment for learning improves student achievement," write Siobhan Leahy, Chistine Lyon, Marnie Thompson, and Dylan Wiliam in "Classroom Assessment: Minute by Minute, Day by Day" (2005, p. 19). The authors cite a research project that found that "teachers who used assessment for learning achieved in six or seven months what would otherwise have taken a year." What's more, write Leahy, and colleagues, "these improvements appeared to be consistent across countries . . . and even held up when we measured student achievement with externally mandated standardized tests" (p. 19).

Reflections ◆ ◆ ◆

Bringing the curriculum to life in the classroom can make math class an exciting place to be. The innovative teacher knows how to create an environment that is safe and engaging—a place in which students are genuinely excited to be discovering the world of mathematics. These innovative teachers have several things in common, including that they

• **Use real-world scenarios that students find interesting.** Math doesn't have to be contrived, these educators agree. The community in which students live abounds with mathematical problems that students can help solve. Relevance leads to motivation. Get over the "When will I ever use this?" wall by making math relevant.

• **Understand the importance of building relationships with their students.** If students are going to learn, they have to be able to take risks—to be willing to discuss their ideas and pose their questions without risk of ridicule. Effective teachers help students feel safe by emphasizing relationships and community. As Don Karlgaard says, "I tell my students frequently, 'We are all friends here, trying to help each other learn mathematics.' "

• **Know that formative assessment—ongoing daily assessment in the classroom—can promote learning.** In the classrooms of effective educators, assessment and instruction are inextricably linked.

4 Nurturing the Struggling Mathematical Mind

Chance favors only the prepared mind.

—*Louis Pasteur, H. Eves,* Return to Mathematical Circles

James Slosson learned from his children how to teach math effectively. "I always struggled with math, but my children struggled with it even more, so I had to become a pretty good math teacher," he says. His experiences as a parent, teacher, and administrator would lead him to eventually create and direct the Math Lab program (*see p. 83*), which is now used in several Washington State schools to help struggling students develop and strengthen positive perceptions of mathematics.

To strengthen positive perceptions is the first of six goals listed on the Math Lab Web site (www.techmathlab.net). It leads the list because Slosson understands how attitude can be everything. Once students *believe* they can do the math in the program, other learning objectives can be obtained, say educators and researchers.

Accentuate the Positive

It seems a natural conclusion to draw: Happiness has a positive effect on children's learning, memory, and social behavior, and that, in turn, boosts students' confidence in their ability to learn and get along with others. Rather than just assume the evident truth of such an observation, however, Tanis Bryan and James Bryan decided to formally assess the impact of positive moods on students' feelings of self-efficacy and

Math Lab

Giving Struggling Students a Second Chance

"Kids who struggle can learn math," asserts James Slosson—and he has spent the last three years proving his point. Slosson created and now teaches Math Lab: Applied and Technical Mathematics, which is designed for students who have learned to dislike math, mainly because they don't get it as easily as other students. "I'm a retired principal of an alternative school," he says. "That provided me with a lot of insight for this program—insight about kids who struggle and in math, particularly."

Slosson used that insight to create a program that combines traditional remedial activities—such as drills on basic skills—with more complex problems that students must solve in multiple steps. "The kids can learn these concepts, so long as you keep the numbers simple," he says. And, because the program focuses on relationships and instruction, as well as conceptual skills, students are more willing to bring their best efforts to class and to test situations, Slosson notes.

The ultimate goal? That those students who complete the Math Lab program will have

- A more positive perception of mathematics (as measured on affective survey, attendance, and parent comments).
- Improved their general math skills (as measured on teacher-made assessments and norm-referenced tests).
- Increased specific knowledge in areas useful in most occupations and other academic areas (as measured by assignment completion data).
- Gained skills in designing and conducting experiments to use mathematics constructively (as measured by in-class test data).
- Improved their scores on high-stakes tests, especially the Washington Assessment of Student Learning (WASL).

- Increased enrollment and success in the regular math continuum (as measured by enrollment follow-up data).

The program has been successful. In the initial pilot year, for example, the general math class went from zero percent passing the math portion of the WASL to 22 percent passing, according to Slosson.

The second and third years (2003–04 and 2004–05) of full implementation took place at a large rural, suburban high school (1,400 students) in Western Washington. For several years in a row, about 25–39 percent of sophomores passed the state high-stakes test.

Now, more than 40 schools in Washington State use Math Lab materials.

More information on Math Lab is available on the Web at www.techmathlab.net.

math performance. Their research suggests that there is power in positive thinking.

Bryan and Bryan conducted two studies, one of elementary children at risk for referral and another of high school-age students with learning disabilities. In each study, one group of students was exposed to “positive mood induction”—they were asked to think about the happiest day of their lives—before being asked how many math problems they could solve, accurately, in a five-minute period. These children were then given five minutes to work on the problems. A second group of children served as the “non-treatment control group” and were given the same tasks as the first without the positive mood induction.

In the first study, Bryan and Bryan found that their hypothesis—that positive mood induction can enhance children’s feelings of self-efficacy and thus enhance math achievement—was “partially supported by the results.” As the authors reported in the *Journal of Learning Disabilities*, “The experience of simply reflecting on some happy event of the past significantly increased math accuracy," but performance accuracy "did not appear to operate through feelings of self-efficacy" (1991, p. 492).

In the second study involving adolescents, however, positive mood induction *did* increase feelings of self-efficacy. "Students in the positive-affect condition estimated they could do more problems than the students in the no-treatment conduction," write Bryan and Bryan. What's more, "students in the positive-affect condition computed more problems accurately than students in the control condition" (p. 492).

According to Bryan and Bryan, the results of the second study are important because "it's expected that children's self-awareness and accuracy in making estimates about their performance are likely to increase with age." This suggests, they write, that "inducing positive mood can have a facilitating impact on adolescents' self-confidence" (p. 493). Additionally, Bryan and Bryan note, because the students tested represented "an enormous range of math achievement levels," the results suggest that "irrespective of initial achievement levels, induction of positive mood affect has a positive influence on adolescents' belief in their self-efficacy" (p. 493).

Other studies subsequently have supported Bryan and Bryan's findings, and, certainly, anecdotal evidence from teachers suggests self-confidence can, indeed, lead to competence in math.

Conversely, many educators note that students won't be academically successful if they lack positive attitudes and perceptions about themselves, their peers, the teacher, and the value of the tasks that make up the lesson. Moreover, "no matter how ambitious and well-articulated a lesson may be, no genuine and lasting learning will occur if your students do not perceive themselves to be valuable," writes John Brown in "Promoting Positive Attitudes About Learning" (2002, p. 7).

Brown, a former teacher and administrator, says there are many ways that educators can help students develop and maintain positive attitudes and perceptions about learning. Teachers can, for example, be enthusiastic about the material they teach. "If you are excited about the content, students will pick up your enthusiasm," Brown writes. Another suggestion: "Don't allow self-defeating and negative self-talk in your classroom," he writes. "Work with students to identify negative self-comments, then help them use positive self-talk and self-acknowledgment" (p. 7).

Boosting Confidence by Honing Self-Awareness

Jason Cushner, who has spent much of his teaching career helping students overcome their math phobias and begin to see themselves "as academically capable, as scholars," agrees that self-talk can be self-fulfilling. "What you say creates a lot of the reality of the world in which you live," he says. Cushner believes that if a student thinks negatively about his performance, he will not perform well. Instead, he will quickly shut down and avoid the math problem rather than work on it. The opposite is also true, he notes. "If a student perceives himself as good in math—even if he has low skills—he will start to figure out a problem as soon as he gets it. He'll say, 'I can do this.' And that creates a positive loop: the student is confident that he can solve the problem; therefore, he engages with it and works to solve it." This, in turn, develops his skills and understanding, says Cushner. He comes to an answer for the problem and takes that confidence to the next problem he encounters. Such an attitude leads to increased competence in math, Cushner believes. "It's easy to work on and improve [students'] skills when they have the work ethic," he observes.

Teachers can do many things to boost middle and high school students' confidence in their ability to learn mathematics. Effective teachers try to understand their students' attitudes about math, for example, and they help students see how they are, in large part, responsible for making math class enjoyable—or a drag.

Nancy Foote has a standard, first-day message for students. "The first thing I tell students is that I have seen all their files and test results. Then I tell them that if I haven't talked to any of them or their parents, then there's no reason to assume that they don't have the ability to succeed in my class." Foote stresses the importance of positive self-talk, telling students that she knows whereof she speaks: When she was in college, she failed a physics exam because, she believes, she set herself up for failure. “I told myself I couldn’t do the problem,” she recalls. “I talked myself into being a loser.”

According to Foote, teachers have to help students learn to focus on success and what they can do well. In an activity she says she learned from Dr. Phil, Foote asks her students to look around the room and see everything that’s blue. She then instructs them to close their eyes. When they open them again, they’re not to look at anything that’s blue. “But, of course, they all see the blue because that’s what they’re thinking about,” Foote says. The exercise is designed to help students understand the power they have over their minds—and that their minds have over them if they aren’t aware of it. As Foote explains to her students: “If you tell yourself you can’t do it, you won’t. . . . Learn to think, instead, that ‘It’s okay if I can’t do all of these problems, because there are a bunch of them I can do.’ ”

And Foote makes sure she practices what she preaches, taking a more positive approach to simple things, like handing back graded assignments. “I make it a point to celebrate successes,” she states. “For example, I never write the number of wrong responses a student may have gotten on a paper. I list the number of problems that are right.”

Maintaining a positive attitude about your own ability to accomplish a task requires a disciplined mind—and that begins with reflection, say many educators. Therefore, the first homework assignment Jeffrey Chaffee gives is a writing task. “I ask my students to write their mathematical autobiographies,” he explains. Chaffee provides a series of prompts that students respond to. Their answers help him gauge his students’ attitudes about math, says Chaffee: “Tell me about a good experience you’ve had in math class. Tell me about a bad experience you’ve had. If you hate math, I want to know that, too.”

Some of the responses Chaffee receives are touching: “One of my earliest math memories is counting apples with my father. I don’t remember why we were doing it, but I remember thinking it was fun,” wrote one student. Others are blunt: “I hate math. I have always hated math. I will always hate math.” Chaffee responds to such negativity by assuring students that by the end of the year, they won’t hate math. “You don’t have to love it,” he states, “but you have to have those basic skills.”

The autobiographies also give students an opportunity to let Chaffee know how they learn best. "Many students in middle school have done the multiple intelligences survey—they know what kind of learners they are," he says. "I try to help kids advocate for their way of learning. Kids need to be able to say, 'I learn . . . by seeing it,' for example." Chaffee notes that teachers should not take such comments as criticism. "It could be that the teacher explained things really well, but not in a way that particular student was comfortable with," he says. Use such comments to guide instruction, instead, Chaffee advises. When students speak up, he says, "they give me insight, and I may do something different in class as a result. If I have kids who tell me that they learn better through hands-on activities, for example, I'll use manipulatives."

Seeking student input, as Chaffee does, is important in creating an open and honest culture in the classroom. Some students' comments, however, may smart. That's why Foote, Chaffee, and other teachers have learned that although they'll solicit input, they'll also remain objective.

It's important not to take students' attitudes about mathematics personally, agrees Don Karlgaard. "The best thing I can do is have a lot of energy and show my kids I that care about them and that I care about the subject—that it's fun and exciting," he says. Some students are more challenging than others, of course, but Karlgaard sees that as an opportunity to help them "realize success and build their confidence." He concedes that some students are "pretty hardcore—they think school sucks." How do you work with someone like that? Karlgaard takes it one day at a time. "I make it a point to talk to them every day, one on one," he says. "I want them to know I have a goal for them—that they're going to be successful. I try to help them see that their negative attitudes are an obstruction to being successful."

Boosting Confidence Through Curriculum and Instruction

As important as it is to help students feel positive and confident about their ability to achieve in mathematics, Slosson cautions that for some students, improving their mindset is not enough. Teachers need to "accept and recognize that some students struggle with math. It doesn't do any good to tell them they would be successful if they just work harder; they know that isn't true," Slosson writes in "When 2 + 2 Doesn't Equal Four," (*Principal Leadership*, 2004). By acknowledging students' "lack of inherent math aptitude," teachers will be in a better position to help them, Slosson notes (p. 47).

A first step is to learn more about how students' minds work and about their personality types, Slosson recommends. Chaffee, for example, asks his students to identify their intelligence strengths so he can gear his instruction accordingly. Other educators may focus on learning styles (*see The Learning Styles-Math Connection, p. 92*). Slosson administers the Myers-Briggs Type Indicator instrument (MBTI), which he first began using when he was a principal at an alternative school.

"After five years of administering the MBTI and collecting data, we noticed that most of our students—92 percent—were Perceivers (P) rather than Judgers (J)," says Slosson, who observes that at typical high schools, the P and J split is usually "about 50–50." (*See What Is MBTI, p. 96.*) He wasn't surprised by his finding, however: Although Js are "all about beating the deadline," Ps, in general, "struggle with deadlines, won't start a project until the pressure is on, and seem to never do their homework"—characteristics that could be used to describe the students at his alternative school, as well as the majority of students enrolled in Slosson's Math Lab (*see Myers-Briggs Results, p. 97*).

Because schools tend to favor Judgers over Perceivers, Sensors (sequential) over Intuitors (random), and Introverts over Extroverts (83 percent of Math Lab students, by the way, are Extroverts), Slosson realized that "everything is rigged against" students who struggle with math. Curriculum and instruction had to change if these students were going to realize any success in math.

So, Slosson designed the Math Lab program to address what he sees as a major flaw in the curriculum that students in learning assistance programs usually receive. "It's all low-level mathematics that focuses on computation," he says. Beyond just being boring, it also sets students up to fail on state mathematics exams. Students who receive the typical remedial math curricula, "never see the multistep problems that show up on [achievement] tests," Slosson states. The remedial curricula should, therefore, be expanded "to include some geometry, probability, graphing, pattern recognition, basic algebra emphasizing the creation of simple equations, and other concepts that are tested," he writes (2004, p. 47). Indeed, Slosson's Math Lab allots 50 percent of the class time to build skills, while the other half of the course focuses on problem-solving activities, selected to match similar problems on the Washington state achievement tests.

Curriculum, of course, is only one side of the coin. "There is not one curriculum you can buy" that can compensate for a poor teacher, Slosson observes. He recommends a highly-interactive instructional approach that emphasizes hands-on learning activities. "De-emphasize direct instruction," Slosson advises. His Math Lab curriculum, for example, features problem-solving activities that are designed for pair or small group work.

Boosting Confidence by Building Community and Trust

Becoming more self-aware is something students can do to advocate for instruction that better meets their needs. Gaining a better understanding their students' minds and personalities is something teachers can do to select complementary curricula and classroom activities. Together, students and teachers can build trust and community in the classroom.

Trust is an important issue in all classrooms—for all students—but it's especially important for those who have had poor experiences in math, says Chaffee. Students, he says, "need to know that it's okay for them to make mistakes, that if they don't put themselves out there, growth won't happen."

Many of the strategies designed to help students become more confident help build trust as well: Karlgaard greets his students, individually, every day. Chaffee asks students for their input. Foote celebrates success. Slosson regularly uses cooperative learning activities. These seemingly small actions are important because they "help students feel accepted by their peers and . . . reinforce the power of group interaction and cooperation," writes Brown (2002, p. 7).

Still, it's impossible to build community and trust if a sense of comfort and order are missing from the classroom, notes Brown, who suggests that teachers "discuss with students the meaning and rationale for rules and procedures," work with students to create those rules and procedures, and then post them in the classroom (2002, p. 7).

It's all about relationships, says Slosson. His Math Lab program promotes relationships between the teacher and the students and among the students themselves. Toward that end, students gather in a circle at the beginning of each class. Over the course of each month, every student gets interviewed while the others listen. Through this activity, "they start to trust one another," says Slosson, and students become more willing to openly admit to their struggles with math. "Suddenly, they're willing to use their fingers for counting more openly," he observes.

Like Foote, Slosson likes to also celebrate success, and he isn't shy about using rewards, such as giving students special math stickers or some other goodie. "I tell students that when they get four assignments done, they can get something from the 'treat' box," he says. It's a practice normally associated with elementary school—and Slosson knows that. Still, most of the students in his Math Lab "never got the treats in elementary school" because they never got the right answer. Now, after so many years of frustrating results, Slosson is making sure his students get the math—and the treats.

Reflections ◆ ◆ ◆

- **Teachers must vary instruction to reach more students.** Understanding how each student learns can help teachers plan lessons that provide different entry points into the content. There are many tools available today that can help teachers become better acquainted with their students' minds. It's important to provide enough variety in lessons and assessments so that all students can engage with the subject matter and demonstrate their understanding.

• **Students can advocate for themselves.** One of the key points educators in this chapter have made is that students need to have a good understanding of how they learn best, and they need to share that knowledge with their teachers. All students—those who excel in math and those who struggle with the subject—should be given opportunities to reflect about the learning process, and teachers can model how to effectively express their learning preferences to others.

The Learning Styles–Math Connection

That some students struggle with and become turned off to mathematics comes as no surprise to Harvey F. Silver and Richard W. Strong, who suggest that teachers too often fail to recognize that "different students have different learning styles and need different things from their math teacher" (Silver & Strong, 2003, pg. 6).

Teachers, for example, need to continually challenge and motivate students who easily comprehend mathematical concepts while also meeting the needs of students who struggle with math.

That may be easier said than done when it comes to mathematics instruction, say Harvey F. Silver and Matthew Perini, education consultants with Silver Strong & Associates. In workshops, many math teachers had difficulty determining how they could incorporate an understanding of learning style into their practice. "Math teachers would say, 'Well, that's nice, but how do you do this in mathematics?' " says Silver.

So Perini and Richard Strong, also with Silver Strong & Associates, decided to conduct an in-class experiment: "We said, 'Let's give students some math problems and collect information on how they solved those problems,' " says Perini. They found, perhaps somewhat obviously, that there were "some strong differences in ways children approach mathematics and how they view mathematics." The experiment revealed that there are essentially four types of math students. Teachers must be aware of these types so they can be sure to plan learning activities that honor students' preferred style, says Perini (*see Understanding Statistics, p. 94*).

Students who want to learn practical information and set procedures are **Mastery** math students. These students like math problems that have been solved before, write Silver and Strong in the introduction to *Styles and Strategies for Teaching High School Mathematics* (2003). These students have the most difficulty when math content becomes highly abstract, says Silver in an interview. "If they have a procedure, they're okay, but if they have to discover the strategy, they have more difficulty." These students want a math teacher who "models new skills, allows time for practice, and builds in feedback and coaching sessions," write Silver and Strong (p. 13).

Interpersonal math students love to learn math through dialogue and group work, says Perini. They like to consider how math helps people and prefer problems that have real-world applications. "These are the students who love to talk to their neighbors and experience difficulty when instruction is focused on seatwork and independent work," Perini explains. These students "need the teacher to pay attention to them and their struggles with math," he says (p. 13).

Understanding math students are interested in the why of math, explains Perini. They like math problems that ask them to take a position. These students "try to find a pattern and they're always on the lookout for questions or little tricks," Perini says. These students have difficulty when collaboration is part of the lesson, he continues. "These students are most frustrated by group work." The math teacher who challenges students to think and requires them to explain their thinking is most popular with these students, write Silver and Strong (2003).

Students who want to use their imagination, that are drawn to projects that allow them to think outside the box, and who tend to visualize problems are **Self-Expressive** math students, says Silver. Self-Expressive students want choice and creativity, he notes, and aren't comfortable in a classroom that emphasizes drill and practice and rote problem solving. What's more, they want teachers who invite creative problem solving into the math classroom, write Silver and Strong (2003).

Awareness of the four types of math students is, of course, a first step. An ideal next step, says Silver, is for teachers to "develop units of instruction that support these learning styles." Unfortunately, he points out, the "overwhelming bias in math instruction is toward one or two styles," adding that the "inability of students to achieve as well as they can in math is as much a style issue as it is a cognitive issue."

Source: Adapted with permission from *Priorities in Practice: The Essentials of Mathematics, K–6*, (pp. 114, 125, 128–129), by K. Checkley, 2006, Alexandria, VA: Association for Supervision and Curriculum Development.

Understanding Statistics

Effective teachers use many strategies to draw students into the content. Creating lessons that honor learning styles is one of those strategies.

In this exercise, created by Silver Strong & Associates to meet Nebraska state math standards for data analysis, probability, and statistical concepts (middle school), students read and interpret a chart that identifies the accuracy with which several basketball players make their free-throw shots.

The Scenario: Statistics are part of everyday life, from political polls to marketing surveys to the complexities of averages and winning percentages in sports. Understanding how statistics are used and how they can help us make good decisions is an important lesson for students. In this exercise, students will analyze data, compare statistics, identify criteria for decision making, and explain their choice statistically.

The Hook: You are the coach of the Stars basketball team. Your team is one point behind and, at the final buzzer, a technical foul was called on your opponent, the Flames. You have to select one of your five players to shoot the two free throws. If the player makes the first free throw, the game goes into overtime. If the player makes both free throws, you win the game. Who would you choose and why?

Player	Free throws during the season	Free throws the last two minutes of the game
Tanya	12 out of 15	1 out of 3
Maria	40 out of 50	18 out of 27
Elizabeth	45 out of 60	12 out of 16
Mora	9 out of 18	4 out of 4
Shanika	27 out of 54	15 out of 25

Mastery Task, Activity 1:

Agree or disagree

- Mora took the most free throws.
- Mora made more free throws than Elizabeth.
- Maria has the highest percentage of successful free throws.

Interpersonal Task, Activity 2:
Who did you choose to shoot the free throw? Tell why.

Understanding Task, Activity 3:
Does the team shoot better during the last two minutes than it does during the season?

What is the overall percentage for the season? For the last two minutes?

What is the difference in the percentage between the free throws made during the season versus the last two minutes?

Self-Expressive Task, Activity 4:
What criteria would you use to choose a shooter? List them.

1. ______________________________

2. ______________________________

3. ______________________________

What factors, other than statistical evidence, would you use to make your choice?

Source: Adapted with permission from Silver Strong & Associates. (2003, March). Thoughtful Education Press. 1-800-962-4432. www.silverstrong.com.

What Is MBTI?

According to Wikipedia, the Myers-Briggs Type Indicator (MBTI) is a personality test that can help people identify their personality preferences. The test, developed by Katharine Cook Briggs and her daughter Isabel Briggs Myers during World War II, identifies 16 different personality profiles.

The personality profiles are derived when people identify their inclinations from among the following four pairs: extraversion or introversion, sensing or intuition, thinking or feeling, and judging or perceiving. Participants are given one of 16 four-letter acronyms, such as ESTJ or INFP, indicating what they prefer.

Source: From Wikipedia: Myers-Briggs Type Indicator. Retrieved March 15, 2006, from http://en.wikipedia.org/wiki/MBTI#The_type_table.

Myers-Briggs Results

Type	Math Lab Students	All High School Students	Math Teachers
Extrovert	83%	75%	34%
Introvert	17%	25%	66%
Sensor (sequential)	37%	75%	56%
Intuitor (random)	63 %	25%	44%
Feeler	68%	50%	61%
Thinker	32%	50%	64%
Judger (finisher)	12%	50%	64%
Perceiver (procrastinator)	88%	50%	36%

Source: From *Math Lab In-Service Manual*, by J. Slosson, 2004. Reprinted with permission.

5 Implications for Professional Development

The student skit at Christmas contained a plaintive line: "Give us Master's exams that our faculty can pass, or give us a faculty that can pass our Master's exams."

—*Paul R. Halmos,* I Want to Be a Mathematician

Javier Gonzales is a man who wears many hats. He's an award-winning mathematics teacher and department chairperson at Pioneer High School in Whittier, California, and he's a school board member for the district that sends students to Pioneer High. Gonzales created the Pioneer Math Academy, which is designed to foster students' mastery and self-confidence in mathematics, and during the school year, he serves as a mentor teacher and the coordinator of Pioneer's gifted and talented education program.

It's no wonder, then, that Gonzales would be among the 13 elementary, middle, and high school science, math, and technology teachers invited to join the National Academies Teacher Advisory Council (TAC; *see p. 99*). Formed in 2002, TAC is designed to give exemplary educators, like Gonzales, a voice in crafting education policy at the local, state, and national level. This includes professional development policy. And with 20-plus years in the classroom and 20-plus years of professional development under his belt, Gonzales has much wisdom to share.

The Teacher Advisory Council

The National Academies established a Teacher Advisory Council (TAC) in 2002 to directly involve active classroom teachers and integrate their "wisdom of practice" in the National Academies' research and other work in education.

The Teacher Advisory Council

• Works with the education research community to develop new research that is informed by and useful to education practitioners.

• Provides advice about how the National Academies can develop reports and recommendations that can be most effectively implemented in schools.

• Offers guidance about how the National Academies can best communicate with teachers and the larger education community in the United States.

The council consists of a core group of 13 teachers representing the sciences, mathematics, technology, and reading literacy, with a teacher–leader staff member who manages the work of the council. These teachers represent elementary, middle, and secondary grade levels, urban and rural settings, as well as diverse regions of the country.

In October 2004, TAC members gathered in Washington, D.C., at the National Academy of Sciences for a two-day workshop that focused on how to make mandatory professional development more meaningful for teachers and more effective in improving practice.

The workshop (National Research Council, 2005) kicked off with a presentation from Anne Lieberman of the Carnegie Foundation for the Advancement of Thinking. Her address focused on three questions: Why is professional development so problematic? What do we know from research and practice? How do we get from here to there?

Lieberman said that there is little connection between professional development and the reality of the classroom. Not only is it divorced from practice and teacher input, she observed, most professional development is "one-size-fits-all"—an experienced teacher of 20 years gets the same in-service workshop as a teacher who is brand new to the profession. This doesn't really make sense, she suggested. Professional development, Lieberman stated, needs to be differentiated if it is to be effective.

Professional development also needs to reflect and honor the fact that teaching is not just a series of techniques, but that what teachers do in the classroom reflects social and cultural perspectives and experience, as well. "People trust their experience more than they trust the evidence," Lieberman stated.

Lieberman suggested that the National Writing Project (NWP) offers a good model of professional development "that we need to fight for." The NWP has incorporated elements of a professional learning community, she said.

Indeed, said Lieberman, what she learned about the NWP helped shape her thinking about professional development, which includes her belief that

- Teachers should be involved in developing a focus for learning that fits the school.
- Teachers should be involved in the leadership.
- Schools must make time for teacher learning, including providing opportunities for study, dialogue, critique, and feedback.
- Schools should differentiate professional development and provide a variety of options to learn.
- Provisions should be in place for mentoring, coaching, and supporting teacher learning over time.

Effective PD: More Content Specific, Please

Effective professional development can't be one-size-fits-all, Gonzales told educators who attended a two-day workshop in October 2004 (*see p. 102*). Gonzales, for example, believes he should be exempt from future district-mandated professional development sessions that focus on literacy, having listened to California's literacy guru several times. "I teach mathematics," Gonzales stated. To become a more effective teacher, therefore, he needs content-specific workshops (National Research Council, 2005, p. 61).

His colleagues agree with him. In a survey of the workshop participants, conducted before they gathered in Washington, D.C., most respondents (96 percent) indicated they would prefer content-specific professional development, but they reported that only 25 percent of mandatory in-service training focuses on the subjects they teach—the sessions, instead, usually present general teaching or classroom management strategies. These may be helpful, but as one TAC member noted, most teachers want—and need—to "talk about physics or math or some other subject" directly related to their practice.

It is ironic, many educators note, that while teachers are called to honor the different ways in which students learn—by differentiating instruction or tailoring activities to better match students' interests, for example—the same practice is rarely extended to adult learners.

"One wonders why the concept of differentiation . . . has been extremely slow in coming to the world of staff development," writes Robby Champion, an independent educational consultant, in her online forum ASCD's Idea Exchange (http://webboard.ascd.org:8080/~PDIdeas). Although she acknowledges that a lack of resources—monetary and human—contribute to the situation, Champion knows there is a "persistent belief that every staff member should have the same staff development content, using the same model or delivery system, and at the same time" (2005, para. 2).

Still, the pressure for immediate improvements in student achievement and school reform has led to a renewed effort to provide varied learning opportunities for teachers through different learning models, Champion notes. "Educational journals now include a plethora of

What Do Teachers Need to Know?

"The quality of mathematics teaching and learning depends on what teachers do with their students, and what teachers can do with their students depends on their knowledge of mathematics," observed authors of the RAND report *Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics Education*. Unfortunately, note the authors, "the nature of the knowledge required for successful teaching of mathematics is poorly specified" (2003, xvi). The panel, therefore, recommended an extensive research and development program that will help educators gain

- A better understanding of the mathematical knowledge teachers need to be effective in the classroom.
- Improved methods for disseminating useful and usable mathematical knowledge to teachers.
- Valid and reliable measures of the mathematical knowledge teachers have. (RAND Mathematics Study Panel, 2003)

According to the panel, information gathered from a "vigorous and critical research, development, and practice community" would go a long way toward "developing teachers' mathematical knowledge in ways that are directly useful" in helping them help their students attain the skills they need for mathematical thinking and problem solving (2003, p. 78).

Source: Adapted with permission from *Priorities in Practice: The Essentials of Mathematics, K–6*, (pp. 136–147), by K. Checkley, 2006, Alexandria, VA: Association for Supervision and Curriculum Development.

success stories involving districts and schools determined to find [ways] to make staff learning more powerful," she writes. In most of those cases, schools and districts "have shifted their approach to staff learning by differentiating it and avoiding one-size-fits-all." As a result, teachers learn more, more effectively, "when provided with a variety of learning options, rather than having one learning style or time frame imposed on them," she writes (para. 4).

For those interested in differentiating staff development to better meet the needs of adult learners, Champion recommends that school leaders and professional development staff:

• **Make multiple contexts available at different times throughout the year.** Learning goals can be addressed "in different contexts to accommodate teachers' varying schedules or learning preferences," writes Champion. "Options might include a yearlong, large-group university course; an online course that learners can complete individually according to their own schedule; a summer institute for an entire faculty; an ad hoc action research group culled from different schools; or an ongoing, biweekly collegial study group," she writes (para. 5).

• **Incorporate pre-assessment and individual goal setting.** Just like their students, adult learners should "determine what they need to learn, what they already know, and what their individual learning goals should be," writes Champion (para. 6). Learning opportunities can then be tailored to fit different participants' needs.

• **Use student work results as a starting point.** By analyzing their students' work, teachers can better determine "what they personally need to learn to help those students succeed," Champion writes (para. 7).

• **Look to experts as a resource in the group.** "Adult learners appreciate being recognized for their expertise," writes Champion (para. 9). Program leaders can pair novice teachers with more experienced teachers, based on experience and content knowledge, for example. Or, teachers experienced in a particular aspect of teaching mathematics, such as using the graphing calculator, can hold workshops for other teachers.

• **Engage adult learners in action planning.** "Action planning can help . . . make learning more personally relevant and, thus, more meaningful," Champion writes. The action planning process, she notes, allows participants to "reflect on what they are learning, sort through and select what they can best use, address potential hurdles, and plan for their next action steps" (para. 10).

Effective PD: Learning from Other Countries

Champion insists that making professional development more relevant to teachers doesn't take a lot of money or even more time—just creativity. Which is a good thing, Gonzales points out, because additional dollars for professional development are hard to come by. Consider the district for which he serves as a school board member.

The district receives approximately $50 million to provide for its 7,000 students. Of that amount, 85 percent is allotted to cover the salaries of all the people who work in the district, 9 percent of it is allotted to facilities so schools are maintained, and about 6 percent is left for programs for the students, parents, and teachers, which includes professional development. The question is how, said Gonzales. With limited resources, how can schools provide the kind of professional development that will help teachers remain effective? "It is not an easy task at all," he observed (National Research Council, 2005, p. 56).

For guidance, U.S. educators—and policymakers—might be wise to study how other countries deliver professional development, Gonzales suggested. "I was very fortunate to travel to Japan in 1997 and visit several schools," he said. While there, he observed that "every single school across the nation worked with the same standards." As a result, said Gonzales, professional development in Japan is based on those standards. And, because teachers have a shared understanding of national learning objectives, teacher learning is more readily embedded into their daily routines. "Their professional development is not necessarily their summer institutes. It's something that happens daily," Gonzales noted (National Research Council, 2005, p. 57).

For example, every morning before classes begin, teachers will get together and review the lesson plan for the day. The teachers know which standards are addressed in the lesson, which test questions are addressed, and so on, Gonzales said. It's only after a group of three or four teachers goes through an entire lesson—taking about 90 minutes—

that teachers will then deliver it to students. “At the end of the day,” stated Gonzales (National Research Council, 57), “they will all come back . . . and they will look at their lesson [and ask], What went right? What went wrong?”

The analysis doesn’t stop there. Teachers’ reflections about the lesson are published on the Web “so that other teachers across the nation” can see best practice at work in the classroom, Gonzales said. The Japanese system, he asserted, shows how adopting national standards makes it much easier for teachers to share the “many beautiful things” they’re doing in the classroom. As a result, all students, learning the same material, benefit, he said (National Research Council, 2005, p. 57).

After that visit, Gonzales was convinced that U.S. schools could be greatly improved by implementing one sweeping systemic change—national standards—and by encouraging more teachers to participate in a specific professional development practice—lesson study (*see About Lesson Study, p. 106*).

“Lesson study is awesome,” says Jason Cushner. The best learning occurs when small collaborative groups of teachers get together to “observe, tweak, and continually refine a lesson” and then support each other in implementing that lesson in the classroom, he states.

It’s a phenomenal practice, agrees Debra Scarpelli, a 7th grade mathematics teacher at Slater Junior High School in Pawtucket, Rhode Island, who regularly meets with three or four of her colleagues, including the special education teacher, to “pick a topic that’s hard to teach and build a lesson together.”

Suppose Scarpelli and the 7th grade teacher team want to develop a lesson that shows why multiplying two negative numbers results in a positive number, for example. The school will use grant money to hire substitutes while the teachers take that class time to work together to create a lesson. “We look at the questions kids might ask and address possible misconceptions,” says Scarpelli. Then, one teacher teaches the lesson while the other teachers “become other sets of eyes” to gauge how students react. Once the lesson is finished, the teacher who taught it discusses her reactions, the observing teachers offer their comments and recommendations, the lesson is modified, and it’s taught again.

About Lesson Study

The term comes from the Japanese word *jugyokenkyuu,* which describes the professional development model that Japanese teachers have perfected (Teachers College, 2002). "During lesson study, a group of teachers researches and writes a lesson plan on a particular theme," writes Ellen R. Delisio in "Lesson Study: Practical Professional Development" (2004). The plan also includes the teachers' expectations for the lesson: How will it help students understand a certain concept better?

"Once the lesson is completed, one teacher from the group volunteers to teach it to his or her class, and the other teachers are given release time to observe the implementation of the lesson and note if and how it met expectations," Delisio writes. Teachers then meet again, "review notes, and decide what revisions are needed," she explains (2004, para. 6).

Source: From *Priorities in Practice: The Essentials of Mathematics, K–6*, (pp. 146–147), by K. Checkley, 2006, Alexandria, VA: Association for Supervision and Curriculum Development.

Teachers will embrace lesson study because it's a collegial practice, absent of judgment, Scarpelli notes. Because the lesson is developed by a group of people, "it's the lesson that's being evaluated—not the teacher." Therefore, it's an excellent professional development approach for less-experienced teachers who "want to learn about the content and how to deal with it" without fear of being compared with their more-experienced colleagues, Scarpelli says.

Lesson study can "strengthen the belief that improvement in teaching is possible," write Catherine Lewis, Rebecca Perry, and Jacqueline Hurd in "A Deeper Look at Lesson Study" (2003/2004, p. 21). One teacher, the authors write, found "that lesson study puts a professional component back in teaching that is generally missing and treats teaching as a science that teachers can analyze and improve" (p. 21).

Lesson study can be effective in helping establish "a community of practice in which teachers routinely share resources and ideas" (p. 20). U.S. educators, however, must understand "that lesson study means far more than just walking through a set of specific activities," write Lewis, Perry, and Hurd (p. 22).

To ensure that the practice remains a viable professional development option, educators must commit to what the authors call a "visible" sign of lesson study: the processes of goal setting, research lesson planning, data collection, discussion, and revision. However, suggest the authors, for lesson study to flourish, educators must also be determined to foster a culture that encourages and enables an ongoing growth of knowledge. This includes providing the "interpersonal resources and motivation" that help ensure instruction will continuously improve, write Lewis, Perry, and Hurd (p. 22).

Effective PD: Learning from Each Other

That lesson study can be effective in motivating teachers shouldn't be surprising—it allows teachers an opportunity to interact with each other professionally and share their knowledge in meaningful ways. Any time two or more teachers gather to review practice and discuss strategies, improvement happens, researchers note.

Take mentoring, for example. Many studies conclude that novice teachers fare better in schools that provide mentors. Mentors support novice teachers by helping them become acquainted with day-to-day school routines and by being sources of advice and inspiration.

Gonzales certainly knows this to be true. He credits his former teacher and mentor at Garfield High School, Jaime Escalante, with helping him remain true to his calling. "When I was confused as to what was to become of my profession, it just took me a few days of sitting in his classroom and observing a master at work for me to be motivated and see the power of teaching," Gonzales said (National Research Council, 2005, p. 60).

Such classroom observations are beneficial for both the teacher being observed and the teacher observing, he added. "There is so much learning to be done," Gonzales said. Then, when educators take time to sit down afterward and have a conversation—that's when reflection truly happens, he said.

If classroom observations are difficult to arrange, Gonzales recommends the next best thing: have a mentor or lead teacher videotape lessons and ask teachers to watch and reflect on their instructional effectiveness.

"I have done this with some of my teachers," said Gonzales. By watching the videotape, teachers see what their strengths—and challenges—are. Does this teacher really answer questions he's posed without first waiting 30 seconds? Check the videotape. Does that teacher call on boys more than girls? Check the videotape. "I think this is some of the most powerful learning that can happen," said Gonzales. The teacher being videotaped stands to gain from the practice, of course; so, too, does the person operating the camera. She has to be ready to respond with encouraging and constructive comments and be ready to "point out some of the important things" the videotaped teacher should consider, Gonzales said (National Research Council, 2005, p. 61).

Peer observations are vital to improving teaching, agrees John Bakelaar. "Anytime I can watch people teaching, I learn," he says. After spending a Friday afternoon watching a fourth-year teacher, for example, Bakelaar came away with some new ideas for teaching students how to add exponents. "It's just amazing how much talent you have in your own building," Bakelaar says.

That talent can be harnessed to conduct on-site professional development, adds Joseph Siddiqui. "Typically, professional development happens within our math department, where we bounce ideas off each other and debrief our lessons together," he says. Occasionally, however, Siddiqui will "set up presentations for other teachers" at the school and around the state. Teaching others, Siddiqui observes, "is good professional development" because he has to have—or has to obtain—a deep understanding of a concept to teach it to others.

Learning best practice from other teachers is the premise behind a unique, after-school math workshop for teachers and students in the Cambridge (Massachusetts) Public Schools. Sungmi Ann-Kim created the Math Club after having spent nearly two decades in the classroom.

"When I was a teacher, I attended many workshops; I saw and learned many interesting concepts. Yet, the challenge was always this: How could I implement into my instructional practice what I had learned in a workshop?" she writes in "The Math Club: Providing Professional Development and Student Enrichment" (2002, p. 5).

When Sungmi left the classroom to become a mathematics staff developer for the bilingual department in the district, what she remembered about her professional development experiences helped shape her ideas for how to transform the workshop experience "and give teachers opportunities to see experienced educators working with real students" (p. 5).

Teachers and students throughout the city volunteered for the workshop. At each of the 10 two-hour sessions, Sungmi led the workshop with 20 to 25 students and approximately 10 teachers. The students were in small groups of two to four and each teacher would sit with and closely observe one group.

Each session had a similar format: The staff developer presented new materials, such as manipulatives, and asked the children to solve a problem. The students in each group worked together to come up with solutions. Students then presented their solutions to the whole group. The teachers could closely observe the small groups or participate if they choose.

During one session, for example, "we explored the concept of tessellations (patterns of shapes that fit together without any gaps)," writes Sungmi (p. 5). Each group of children was given sheets with equilateral triangles, with each large equilateral triangle made up of nine smaller equilateral triangles. Students colored each large equilateral triangle the same way, using a pattern that the group chose. They then cut out the large triangles and glued them to another large sheet of paper. The students soon saw a consistent pattern. The children demonstrated translation, mirror, point, and rotational symmetries.

"After looking through the students' work, I asked the children to anonymously rate how advanced each group's work was. Those at the more advanced developmental stages were able to figure out the relative differences between the products, but those at less advanced stages were just guessing," Sungmi notes. "The teachers saw all the children's responses and observed the children's developmental stages. They could also see how other teachers and the staff developer worked with the children" (2002, p. 5). The learning did not stop with the workshop, however. After each session, the staff developer visited each of the classrooms and provided follow-up.

The Math Club worked, writes Sungmi, "because there was a perceived need among teachers for staff development." The club was also successful because "the teachers were freed from all constraints and were only there to learn, observe, and experiment; they saw various teaching strategies and how children responded to them," she states (2002, p. 5).

As a result, "Math Club teachers are now confident in their ability to use new materials and instructional approaches because they've worked with the children and have seen how they responded," writes Sungmi. "They are more prepared to use new instructional methods in the classroom because they've watched their peers using those techniques" (2002, p. 8).

Whether in organized workshops, such as the Math Club, or through classroom observations and lesson study, teachers say that what they learn by watching their colleagues is invaluable—worth every minute spent. "It's just unbelievably important," says Jeffery Chaffee. "Everybody has little things they do well and teachers would do so much better if they were given time to observe each other." He adds that his entire approach to teaching changed as a result of what he learned from the teachers he observed.

When he saw how other teachers managed a student-centered, active classroom, for example, Chaffee "became more of a risk taker. I once needed my room to be a quiet, orderly place. Then, I saw how other teachers successfully ran a noisy but productive class. I realized that learning was happening in those classrooms, and that allowed me to let go a little bit. I'm now a better teacher," Chaffee say, "because I've had time to be reflective and to watch others teach."

Reflections ◆ ◆ ◆

• **Is it time for a national curriculum?** In Chapter 2, an argument in favor of national standards focused on curricular issues: what should students learn, when, and to what depth should a topic be addressed? As long as states have different standards, these questions will always have different answers, depending on what part of the United States you happen to be in, say proponents. In this chapter, an interesting twist was added to the argument: a national curriculum would make it easier to create professional development programs that show how creative teachers help students meet those standards. These examples of effective teaching, say proponents, would then be applicable to many more teachers who, in turn, would then help more students attain important learning goals.

• **Show me.** To paraphrase Eliza Doolittle: "Don't say how . . . show me!" Actions often speak louder than words—in Doolittle's arena and in professional development. Again and again, teachers say that watching other effective teachers at work is the best way to enhance their teaching and boost students' learning.

Looking Forward

There is enough research out there on what is good in math education. I feel that we're spinning our wheels creating more, good curriculum. We have the product, now we have to create demand for it. We know what we value, how do we help others value it, too?

—Jason Cushner, in an interview with ASCD, 2005

It's a long and winding road to education reform and those in the trenches, they do get weary.

Imagine, for example, the pleasure one might take in the news that U.S. 4th and 8th graders exceeded international averages in mathematics on the 2003 Trends in International Mathematics and Sciences Study (TIMSS). If you look a little closer, however, you'll see that although scores went up a bit, the relative position was still the same—and students from a greater number of developing countries participated in the latest TIMSS.

This kind of good news–bad news scenario is something William Schmidt deals with all the time. He doesn't mean to be pessimistic when he points out the cloud in the silver lining; he's just being forthright. The TIMSS results are "encouraging on one level," Schmidt says, "until you look at the fact that other countries' scores went up, too."

Schmidt isn't surprised by other international comparisons that suggest there's still much work to do. "What have we done that is fundamentally different that would have changed [the results] between 1995 and 2005?" he asks. He acknowledges "small pockets of improvement" in rigor, depth, and coherence—states have paid attention to the "mile-wide and inch-deep" critique, for example, and have whittled down the number of topics covered. Still, Schmidt says, "whether it's been done centrally enough or big enough" has yet to be seen.

Ironically, although students in other countries, such as Japan and South Korea, fared better than their U.S. counterparts in the mathematics literacy and problem-solving portions of the Program for International Student Assessment (PISA), U.S. students had more confidence in their ability to do the math, reports *Education Week*'s Sean Cavanaugh (2005).

This youthful optimism is one of the reasons that educators, like Schmidt, keep on going. He is currently involved in a five-year project with the National Science Foundation (NSF) to improve mathematics and science teaching and learning in grades K–16. The research and development project, called PROM/SE (Promoting Rigorous Outcomes in Mathematics and Science Education), has "taken the basic ideas learned in TIMSS and other research about coherence and putting together trajectories for content coverage for both math and science," Schmidt explains.

The research effort involves collecting and analyzing data to determine which curricula and instructional approaches prove most effective with the roughly 350,000 students who will initially be involved in the project. (Around 37 percent of these students are urban and rural poor.) "The research is quite extensive," Schmidt says, noting that, in addition to that from the students, data will be collected from 62 school districts and 5,000 in-service and 800 pre-service teachers. The data the project members collect will help them determine students' strengths and weaknesses, as well as what kind of professional development is necessary.

Fortunately, once the five years are up, other schools and students will benefit. The results will be disseminated so others can implement similar reforms. The project is replicable—and it was designed that way. "One of the things that NSF is interested in is what you can scale up and what you can't," and what works for PROM/SE schools, says Schmidt, "can work in other districts, too."

In His Own Words
On NCLB

Another limit to student and teacher achievement is recent legislation and regulations, which place too much emphasis on testing and not enough on learning. There certainly needs to be accountability for federal funds spent on education. Overall, however, the No Child Left Behind Act is hindering education more than it is helping it. I personally have had to curtail the type of teaching that earned me recognition as a finalist for the Presidential Award.

When I was in an independent school free from federal legislation, my courses were innovative and my students soared. Now, I am constrained by testing and have had to spend the last four weeks in my classroom teaching a test preparation curriculum designed by the *Princeton Review*. They were paid handsomely by my school district, an urban school district without enough money for textbooks. We nonetheless feel forced to [allocate] resources to test prep rather than instruction because the tests are high stakes for the school district retaining control over innovative programs in which we believe. There are far better ways to provide exceptional education for all and to create a professional environment in which teachers are held to high expectations for classroom instruction and development of practice.

Source: From "The 2003 Presidential Awardees for Excellence in Mathematics and Science Teaching: A Lesson Plan for Success," [Jason Cushner's testimony for congressional hearing], retrieved January 31, 2006, from www.house.gov/science/hearings /full04/mar18/cushner.htm.

The NSF provided a significant amount of money to fund PROM/SE, an act that is entirely appropriate and the kind of support government entities should lend to education reform, suggests Cushner. As one of four teachers to testify before the House Committee on Science in March

2004, Cushner was asked to bring the representatives up-to-date on the state of mathematics education.

In his remarks, Cushner said that "the single most important step the federal government should take to improve math and science education is to sponsor small teacher groups working together to improve practice" (para. 8). Government policies should support structures that allow teachers to work together to develop and refine their curriculum and instruction, he said.

One model initiative Cushner emphasized is sponsored by the Colorado Council of Teachers of Mathematics and is based on TIMSS research:

> In this curriculum development model—which is the established norm in Japan—teachers work in small groups to develop curriculum and practice. Small groups of teachers convene to define a particular issue or problem they share in instruction (for example, ensuring that students understand the applications of logarithms). Then they work together to design lessons addressing that issue. Finally, all the teachers in the group pilot the new unit in their own classrooms, periodically observing each other's practice to critique the unit and refine it.
>
> This "teachers as researchers" model allows teachers to address the issues that are truly present in their classrooms and curricula. It also keeps teachers fresh and creative, giving them regular [opportunities] to practice the thinking and problem-solving skills necessary for good math and good teaching. Moreover this model incorporates the type of pre-service and in-service training I have found to be most helpful as a teacher: getting plenty of classroom time with plenty of observation and feedback. This also holds teachers accountable to their peers for delivering quality learning in their classrooms.
>
> Contrary to popular belief, salary is not the primary factor keeping good math teachers away from schools. Rather, it is

> the lack of professional stimulation. I personally have spent so much time participating in "professional development" workshops where outsiders to my field come to tell me what to do. I am happy to learn from others, but I often find that these "experts" are unable to give me information that I can use in my classroom. A far more effective use of my time would be to work with my colleagues to develop our practice in such a way as to respond to the real needs of our students. (paras. 8, 9, & 6, respectively)

Much of Cushner's testimony (read his remarks in full at www.house.gov/science/hearings/full04/mar18/cushner.htm) echo what other award-winning teachers and education experts have said in this book about how to best support exemplary teaching and instill in students a profound understanding of mathematical concepts. What's needed? A curriculum that helps students see how math is relevant to their lives, instruction that encourages students to actively participate in the problem-solving process, and professional development that allows teachers to learn from the best source of information: other experienced and effective teachers.

To rephrase an old adage: There's a way. Is there a will?

And the beat goes on.

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Resources

Educational Leadership

Assessment to Promote Learning, November 2005
"Looking at How Students Reason," by Marilyn Burns. *This article focuses on the assessment in teaching used by mathematics teachers to determine whether the lesson was accessible to all students, what the students learned, and how one can improve the lesson to make it more effective.*

The Adolescent Learner, April 2005
"The Self-Paced Student," by Angela Vaughn. *The author details the experience of a high school mathematics teacher who dared to let her students take control of their own learning and, in the process, raised student achievement*.

Learning from Urban Schools, March 2005
"Meeting a Math Achievement Crisis," by Lenora Jennings and Lori Likis. *This article shows how a Massachusetts school turned around its dismal math performance*.

Closing Achievement Gaps, November 2004
"Why Do Students Drop Advanced Mathematics?" by Ilana Horn. *Here, the author examines what schools and teachers can do to halt attrition from higher math.*

Writing!, October 2004
"Writing in Math," by Marilyn Burns. *This article details mathematics assignments in which students' writing leads to clear thinking and deep learning.*

Teaching for Meaning, September 2004
"Research Matters/Teach Mathematics Right the First Time," by Steve Leinwand and Steve Fleischman. *The familiar adage, "Yours is not to reason why, just invert and multiply," may not enhance the mathematical performance of many students.*

Improving Achievement in Math and Science, February 2004
This issue offers suggestions for ways to improve the teaching of mathematics and science, thereby enlarging the number of mathematically and scientifically literate citizens.

Equity and Opportunity, December 2002/January 2003
"Raising Minority Achievement in Science and Math," by Freeman A. Hrabowski, III. *The author suggests ways to encourage more minority students to study science, mathematics, and engineering*.

Reading and Writing in the Content Areas, November 2002
"Advanced Math? Write!" by Sister M. Luka Brandenburg. *A high school teacher finds that students learn calculus better when they write about it*.

"Teaching Reading in Mathematics and Science," by Mary Lee Barton, Clare Heidema, and Deborah Jordan. *Science and math texts often present special difficulties for students.*

Newsletters

Curriculum Technology Quarterly, Spring 2004

"Technology for the Math Classroom"

"Taking the Square Route to Positive Math Attitudes"

Classroom Leadership, March 2004

"Improving Achievement in Math and Science"

Curriculum Update, Fall 2003

"Math Teachers Draw on International Expertise"

"Technology Tools Help Math Learners Visualize, Make Connections"

ResearchBrief

"The Effect of Retaining Kindergarten Students on Reading and Mathematics Achievement," November 28, 2005

"Journal Writing in Mathematics Education," June 28, 2005

"The Effect of Teacher Pedagogical Beliefs on Student Achievement in Elementary-Level Mathematics," March 29, 2005

"The Effects on Adolescent Girls of a Girls-Only Math and Science Curriculum," August 3, 2004

"Teacher Quality Measures and Student Achievement in Mathematics," July 20, 2004

"Mathematics Instruction in the United States," October 28, 2003

Books

Administrator's Guide: How to Support and Improve Mathematics Education in Your School, by Amy Mirra

The Beginning School Mathematics Project (BSM), by Anne McKinnon and Don Miller

Math Wonders to Inspire Teachers and Students, by Alfred S. Posamentier

The Mathematics Program Improvement Review: A Comprehensive Evaluation Process for K–12 Schools, by Ron Pelfrey

Priorities in Practice: The Essentials of Mathematics K–6: Effective Curriculum, Instruction, and Assessment, by Kathy Checkley

Teaching Children Who Struggle with Mathematics: A Systematic Approach to Analysis and Correction, by Helene J. Sherman, Lloyd I. Richardson, and George J. Yard. (Prentice Hall)

Teaching Reading in Mathematics, 2nd Edition (A Supplement to *Teaching Reading in the Content Areas,* 2nd Edition), by Mary Lee Barton and Clare Heidema

Videos

The Brain and Mathematics Series, Tapes 1 and 2

The Lesson Collection, Tapes 17–24: Math Strategies

The Lesson Collection, Tape 33: Mathematics—Words in Context (Intermediate)

The Lesson Collection, Tape 40: Precalculus (Sines and Cosines) (High School)

Audios

Improving Mathematics Instruction Through Coaching, by Glenda Copeland

It's Cool to Be Smart in Mathematics, by Daniel Moirao

Math and Science Revolution: New Teaching Methods That Work, by Tracy Severns

Reading Strategies for the Math Classroom, by Rachel Billmeyer

Three Steps to Math and Reading Success, by Lauren Armour

PD Online

Middle School Mathematics, by Diane L. Jackson, Course Instructor, with Patrick Bathras, Practioner's Corner.
The course addresses the role of the National Council of Teachers of Mathematics in promoting best practices, underscoring the belief that mathematics teaching can improve if educators emphasize understanding rather than simply processes, support relevant activities and lessons, use brain-based strategies to facilitate learning, and combine the use of standards with a sense of mathematical wonder. See a sample lesson at http://pdonline.ascd.org/pd_demo/table_c.cfm?SID=61.

Web Resources

America Mathematical Society
www.ams.org/mathweb/mi-sao.html
The American Mathematical Society site offers information on a host of international societies, associations, and organizations on the Web.

Classroom Compass
www.sedl.org/scimast/resources/cc.html
On this site, ideas, activities, and resources link to instructional concepts in math and science.

The Consortium for Mathematics and Its Applications
www.comap.com
This nonprofit organization's site aims to improve mathematics education for students of all ages.

Educational Standards and Curriculum Frameworks for Math
http://edstandards.org/StSu/Math.html
This collection of resources contains information on national and state standards in math and other subjects.

Educational Technology: Technology As a Teaching Tool
www.enc.org/topics/edtech/classroom
This site includes several articles from the Eisenhower Clearinghouse on using technology to teach math concepts.

ENC Focus
www.enc.org/features/focus/audience/0,6779,1,00.shtm
Articles for math teachers from the electronic magazine of the Eisenhower National Clearinghouse (ENC) for Mathematics and Science Education can be found on this site.

LD In Depth: Math Skills
www.ldonline.org/ld_indepth/math_skills/math-skills.html
Math Skills is a subsection of LD Online that provides information for parents and educators dealing with math-related disabilities.

The K–12 Mathematics Curriculum Center
www2.edc.org/mcc
This site provides information on building an effective mathematics education program using curriculum materials developed in response to the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics.

Lesson Study Research Group
www.tc.edu/lessonstudy/index.html
The Lesson Study Research Group site offers an explanation of lesson study in the United States.

A Maths Dictionary for Kids
www.amathsdictionaryforkids.com
This animated, interactive dictionary for kids explains, in simple language, over 500 common mathematical terms.

The Math Forum: Teacher's Place
http://mathforum.org/teachers
Maintained by Drexel University, this site offers comprehensive resources for math teachers at all levels. Drexel's math education index page (http://mathforum.org/mathed/index.html) has subtopics such as new directions and issues in pedagogy, organizations and journals for math education, research in math education, and technology in math education.

Math-Teach
http://mathforum.org/kb/forum.jspa?forumID=206
This list, established by Drexel University, facilitates the discussion of teaching mathematics.

Mathematical Proficiency for All Students
www4.nas.edu/onpi/webextra.nsf/web/proficiency
This site links to Adding It Up: Helping Children Learn Mathematics, *a 2001 report from the National Research Council that explores preK–8th grade math learning and how teaching, curricula, and teacher education can better support student learning.*

Mid-continent Research for Education and Learning
www.mcrel.org/compendium/Standard.asp?SubjectID=1
This site offers a compendium of K–12 mathematics content standards.

Pathways to School Improvement: Mathematics
www.ncrel.org/sdrs/areas/ma0cont.htm
Provided by Learning Points Associates, critical issues for this area include locating, using, and integrating internet-based mathematics materials; providing hands-on, minds-on, and authentic learning experiences in mathematics; and aligning and articulating standards across the curriculum.

Principles and Standards for School Mathematics
http://standards.nctm.org/index.htm
This site includes the electronic edition of Principles and Standards for School Mathematics, *published by the National Council of Teachers of Mathematics. It includes a collection of related resources to help improve the teaching and learning of mathematics. A login is required.*

PROM/SE
http://promse.msu.edu/default.asp
The PROM/SE site describes a comprehensive research and development effort to improve mathematics and science teaching and learning in grades K–12, based on assessment of students and teachers, improvement of standards and frameworks, and capacity building with teachers and administrators.

Secondary Mathematics
www.internet4classrooms.com/math_sec.htm
This site lists a plethora of Web sites that offer everything from lesson ideas, to lesson plans, to tips on how to use technology.

TIMSS 2003
http://isc.bc.edu/timss2003.html
The site for Trends in International Mathematics and Science Study (TIMSS) compiles data from years of monitoring international progress and declines in math and sciences proficiency. TIMSS data is ideal for framing policy development and accountability measurement. You can preview TIMSS 2003 highlights on the National Council for Education Statistic's Web site at http://nces.ed.gov/pubs2005/2005005.pdf.

TODOS: Mathematics for All
www.todos-math.org
The TODOS site aims to promote equitable and high-quality mathematics instruction for all students, especially Latino and Latina students, through teacher resources.

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Education

The Essentials of Mathematics, Grades 7–12

Effective Curriculum, Instruction, and Assessment

One of the biggest challenges facing math educators today is preparing students for the unknown. How much math knowledge is enough for students who must be ready to contribute to industries yet born through technologies yet created?

Although the "how much" question still inspires debate, educators agree that failing to improve mathematics instruction threatens too many students' futures.

In this book, therefore, award-winning mathematics teachers and respected math researchers share their perspectives on how to best improve the mathematics education all students receive.

- Teachers show how they are striving for equity. The stakes are high and every student needs to attain a higher level of mathematics understanding. The exemplary educators featured in this book share some proven strategies for helping them do so.
- Teachers share innovative lessons that address standards and help students see how math can be used in the world.
- Education experts discuss the research that influences how curriculum is developed and how instructional choices are made.
- Teachers and educators explore ways to vary instruction to meet their students' unique learning needs.
- Professional development experts, including teachers, discuss the kinds of learning experiences that teachers want and need.

We hope the ideas and perspectives you read about in this book will inspire you and aid you in helping students achieve a profound understanding of mathematical concepts.

Association for Supervision and
Curriculum Development

PRIORITIES *in* PRACTICE

THE ESSENTIALS OF MATHEMATICS, GRADES 7–12 ■ ■ ■ ■ CHECKLEY